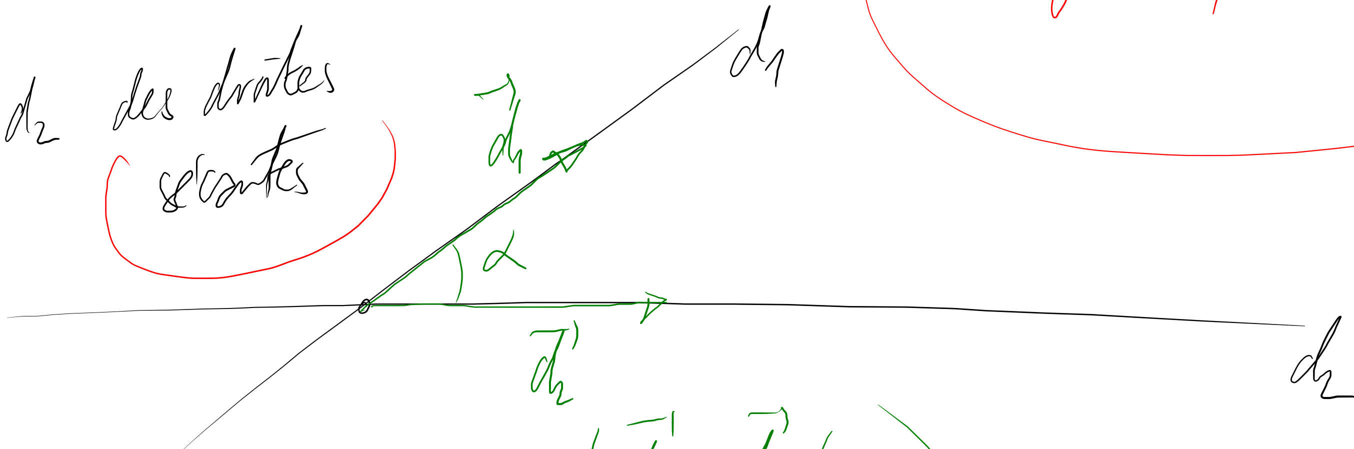
Problèmes métriques

(Angles / distances)

d_1, d_2 des droites sécantes

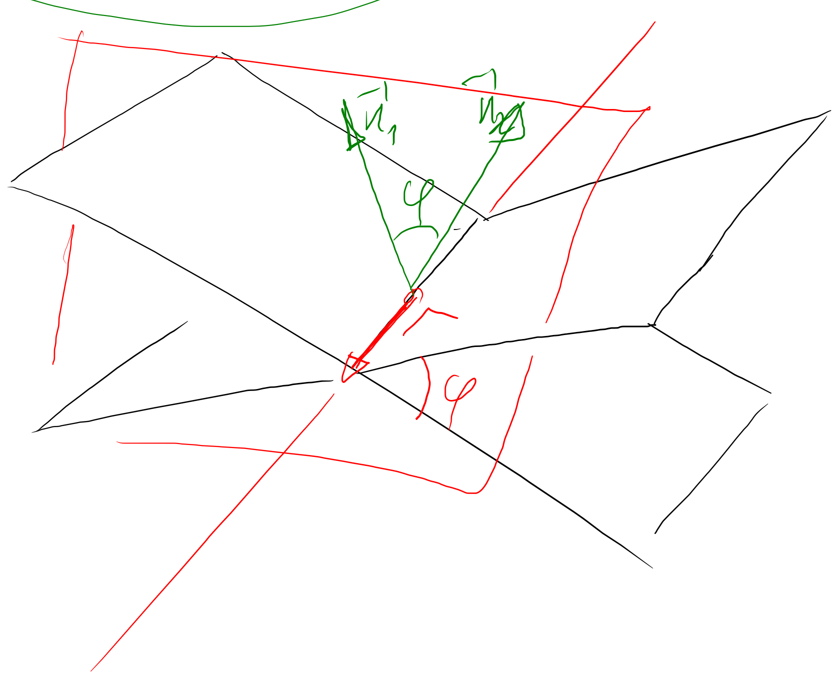


$$\cos\alpha = \frac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\| \cdot \|\vec{d_2}\|}$$

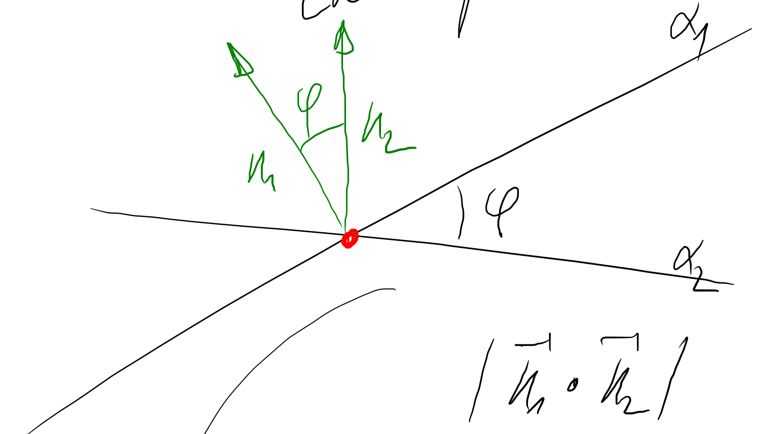
(angle aigu)

α_1, α_2 deux plans

$\vec{n}_1, \vec{n}_2$ les vecteurs normaux correspondants

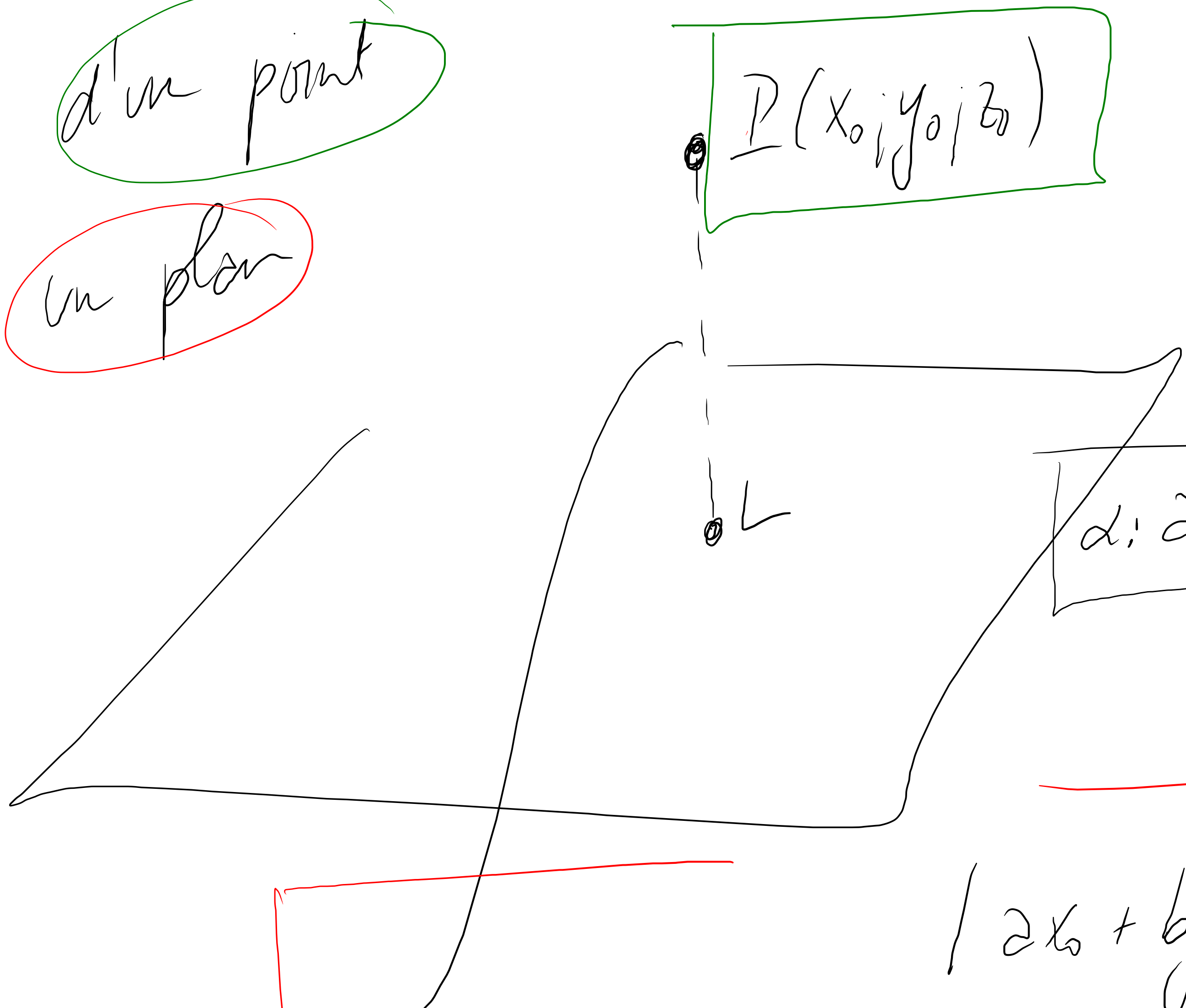


En coupe



$$\cos\varphi = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \, \|\vec{n}_2\|}$$

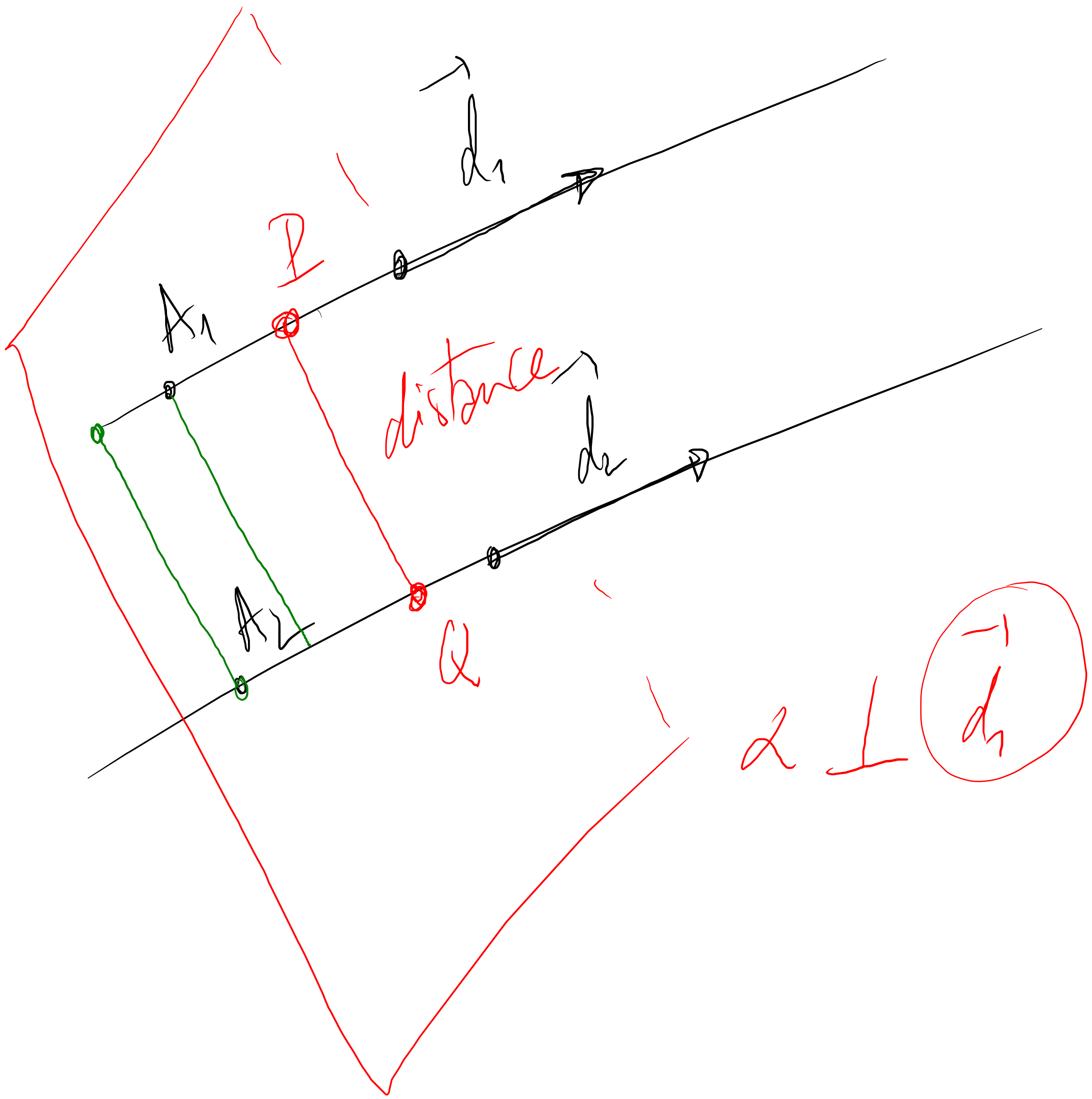
Distance d'un point à un plan



$$\alpha : ax + by + cz + d = 0$$

$$\boxed{\operatorname{dist}(P;\alpha) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}}$$

Distance entre 2 droites :

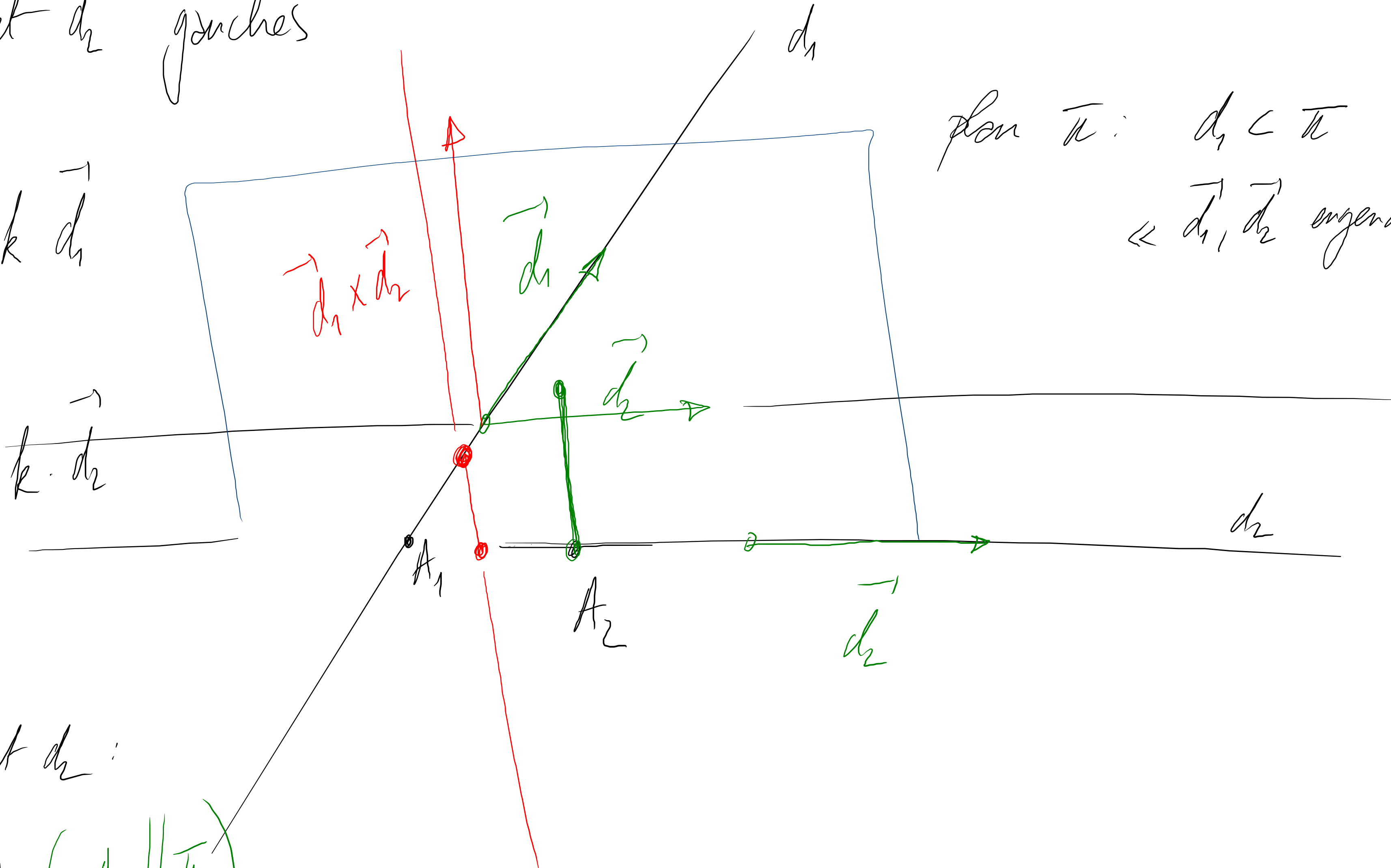


$\|\vec{PQ}\|$ est la distance cherchée.

d_1 et d_2 gauches

$$d_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{OA_1} + k\,\vec{d_1}$$

$$d_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{OA_2} + k \cdot \vec{d_2}$$



plan π : $d_1 \subset \pi$

« $\vec{d_1}, \vec{d_2}$ engendrent π »

distance entre d_1 et d_2 :

distance $(A_2 ; \pi)$ $\quad (d_2 \parallel \pi)$

$$d_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$d_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \ell \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

d_1, d_2 gauches

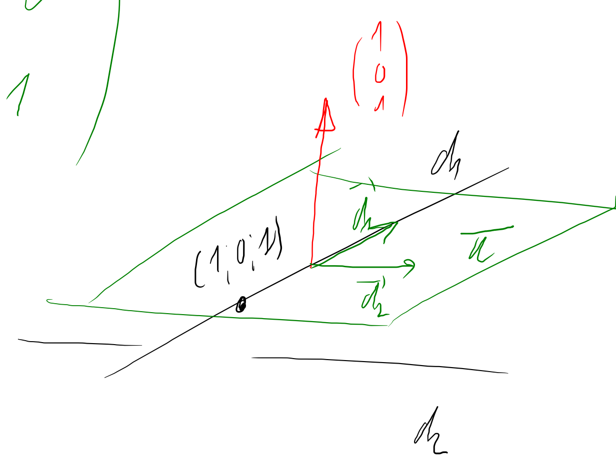
$$\boxed{\begin{array}{l} 1+k = 1-\ell \\ k = -\ell \\ k = 1 \;/\; \ell = -1 \\ 1-k = \ell \\ 0 = \ell \quad \lightning \end{array}}$$

$$\vec{n} \;/\; \pi \;/\; dist\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \pi\right)$$

$$\begin{vmatrix} i & 1 & -1 \\ j & 1 & 0 \\ k & -1 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\pi : x + z + k = 0$ par $(1;0;1)$

$$\boxed{\pi : x + z - 2 = 0}$$



$$dist(d_1; d_2) = \frac{|1+0-2|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \simeq 0{,}707$$

$$\boxed{x-1=\frac{y+1}{-2}=\frac{z}{6}}$$

$$\begin{cases} x-1=\dfrac{y+1}{-2} \\ x-1=\dfrac{z}{6} \end{cases} \qquad \begin{aligned} -2x+2&=y+1 \\ 6x-6&=z \end{aligned} \qquad \begin{cases} 2x+y=1 \\ 6x-z=6 \end{cases}$$

$L_2 \leftarrow L_2 - 3L_1$

$$\begin{cases} 2x+y \quad =1 \\ -3y-z=3 \end{cases} \qquad \begin{cases} x=-\frac{1}{2}y+\frac{1}{2} \\ y=-\frac{1}{3}z-1 \\ z=z \end{cases}$$

$$x=\frac{1}{6}z+1$$

$$y=-\frac{1}{3}z-1$$

$$z=1\cdot z+0$$

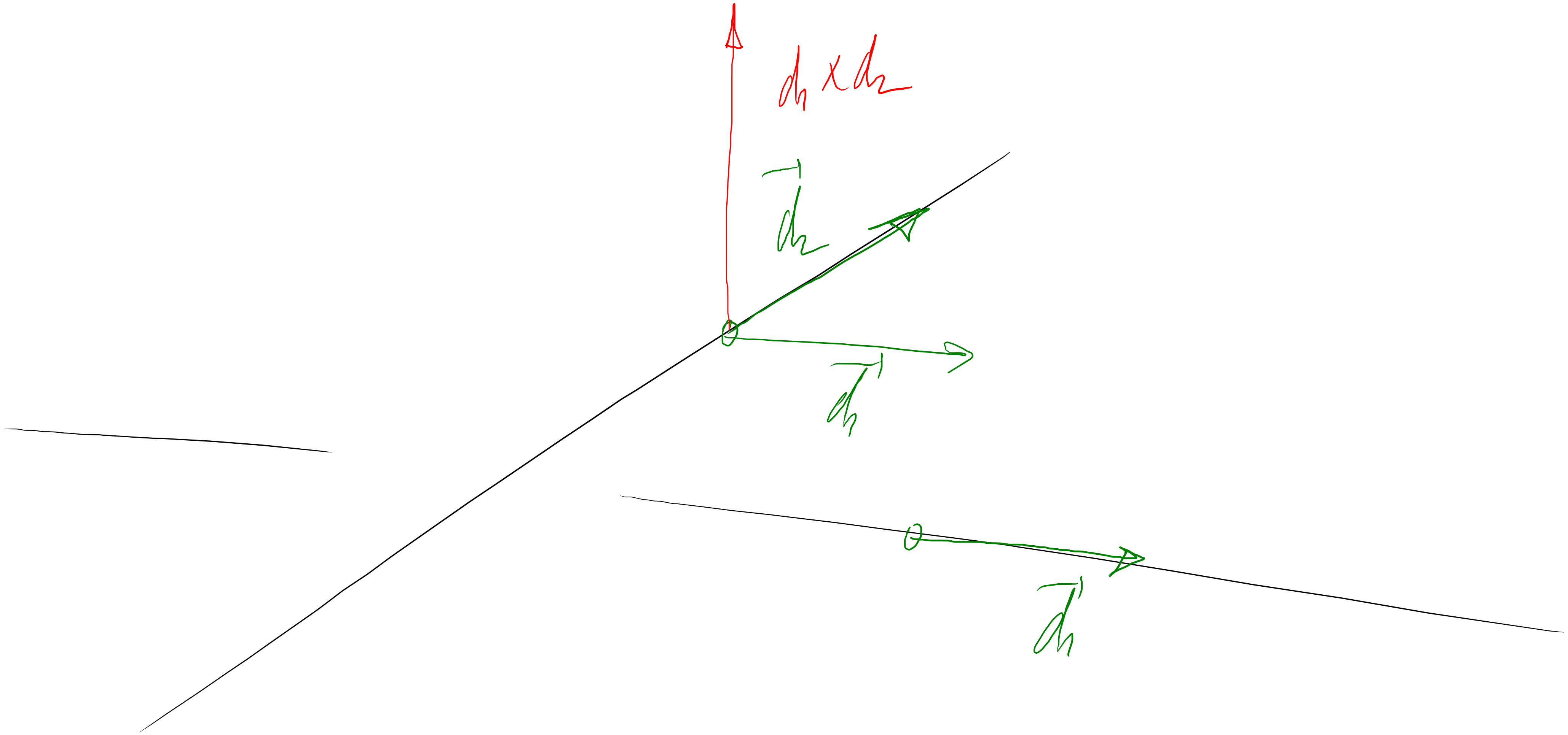
$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}+k\underbrace{\begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix}}_{\vec{d}}}$$

$$\alpha: 2x+3y+z-1=0 \qquad \vec{n}=\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad I=(2;-3;6)$$

$$2(1+k)+3(-1-2k)+6k-1=0$$

$$2+2k-3-\cancel{6k}+\cancel{6k}-1=0$$

$$2k=4-2 \;/\; k=1$$


$\vec{d}_1 \times \vec{d}_2$
$\vec{d}_2$
$\vec{d}_1$
$\vec{d}_1$

$$\begin{cases} x - 2y + z = 7 \\ 2x + y - z = -2 \\ x - 3y + 2z = 11 \end{cases} \quad \begin{matrix} -2 \\ 1 \\ \ \end{matrix}$$

Matrice augmentée

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ \cancel{2}\ 0 & 1 & -1 & -2 \\ \cancel{1}\ 0 & \cancel{-3}\ 0 & 2 & 11 \end{pmatrix}$$

$L_2 \leftarrow L_2 - 2L_1$

$L_3 \leftarrow L_3 - L_1$

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 5 & -3 & -16 \\ 0 & -1 & 1 & 4 \end{pmatrix}$$

$L_3 \leftarrow -L_3$

$L_3 \leftrightarrow L_2$

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -1 & -4 \\ 0 & 5 & -3 & -16 \end{pmatrix}$$

$L_3 \leftarrow L_3 - 5L_2$

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 2 & 4 \end{pmatrix} \longrightarrow \begin{cases} x - 2y + z = 7 \\ y - z = -4 \\ 2z = 4 \end{cases} \qquad \begin{cases} x = 1 \\ y = -2 \\ z = 2 \end{cases}$$

$$\boxed{\begin{array}{l} 3x - 2y + 5z - 4 = 0 \\ 3x + 2y + 5z - 4 = 0 \end{array}}$$

1 variable libre

$x = 1$

$-2y + 5z - 1 = 0$

$2y + 5z - 1 = 0$

$10z - 2 = 0 \qquad z = \frac{1}{5}$