Dans $\mathbb{C}$, $z^2 = a+bi$ admet 2 solutions

si $a+bi \neq 0$

$z^2 = 1+i$ $\quad \boxed{z = x+yi}$ $\quad x, y \in \mathbb{R}$

$(x+yi)^2 = 1+i$ $\quad |x+yi|^2 = |(x+yi)^2| = x^2+y^2 = |1+i|$

$= \sqrt{2}$

$x^2+y^2 = \sqrt{2}$

$x^2 + 2xyi + (yi)^2 = \boxed{1+i}$

$x^2 + 2xyi - y^2 = 1+i$

$(x^2-y^2) + (2xy)i = 1+1i$

$$\left\{\begin{array}{ll} \boxed{x^2-y^2=1} & L_1 \\ \boxed{2xy=1} & L_2 \longrightarrow y = \frac{1}{2x} \\ \boxed{x^2+y^2=\sqrt{2}} & L_3 \end{array}\right.$$

L_1+L_3 : $2x^2 = 1+\sqrt{2}$

$x^2 = \frac{1+\sqrt{2}}{2}$ $\quad x = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$

$$(x+yi)^2 = a+bi \qquad |i| = 1$$

$$\begin{cases} x^2+y^2 = 1 = \sqrt{a^2+b^2} \\ x^2-y^2 = 0 = a \\ 2xy = 1 = b \end{cases}$$

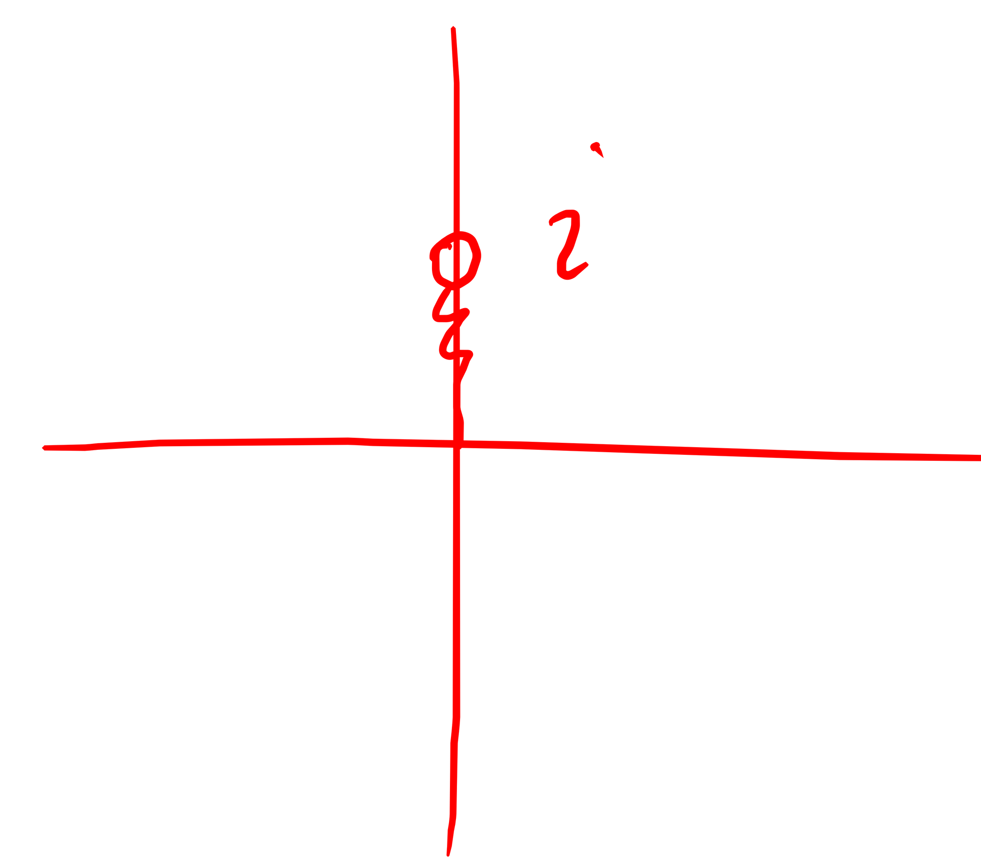
$$(x+yi)^2 = i = 0+1i$$

$$|(x+yi)^2| = |i|$$

$$|x+yi|^2 = |i|$$

$$\left(\sqrt{x^2+y^2}\right)^2 = 1$$

$$x^2+y^2 = 1$$



$z_1 = 3i$

car $(\pm 3i)^2 = 9\, i^2 = -9$

$z_2 = -3i$

$x^2 + y^2 = 9$

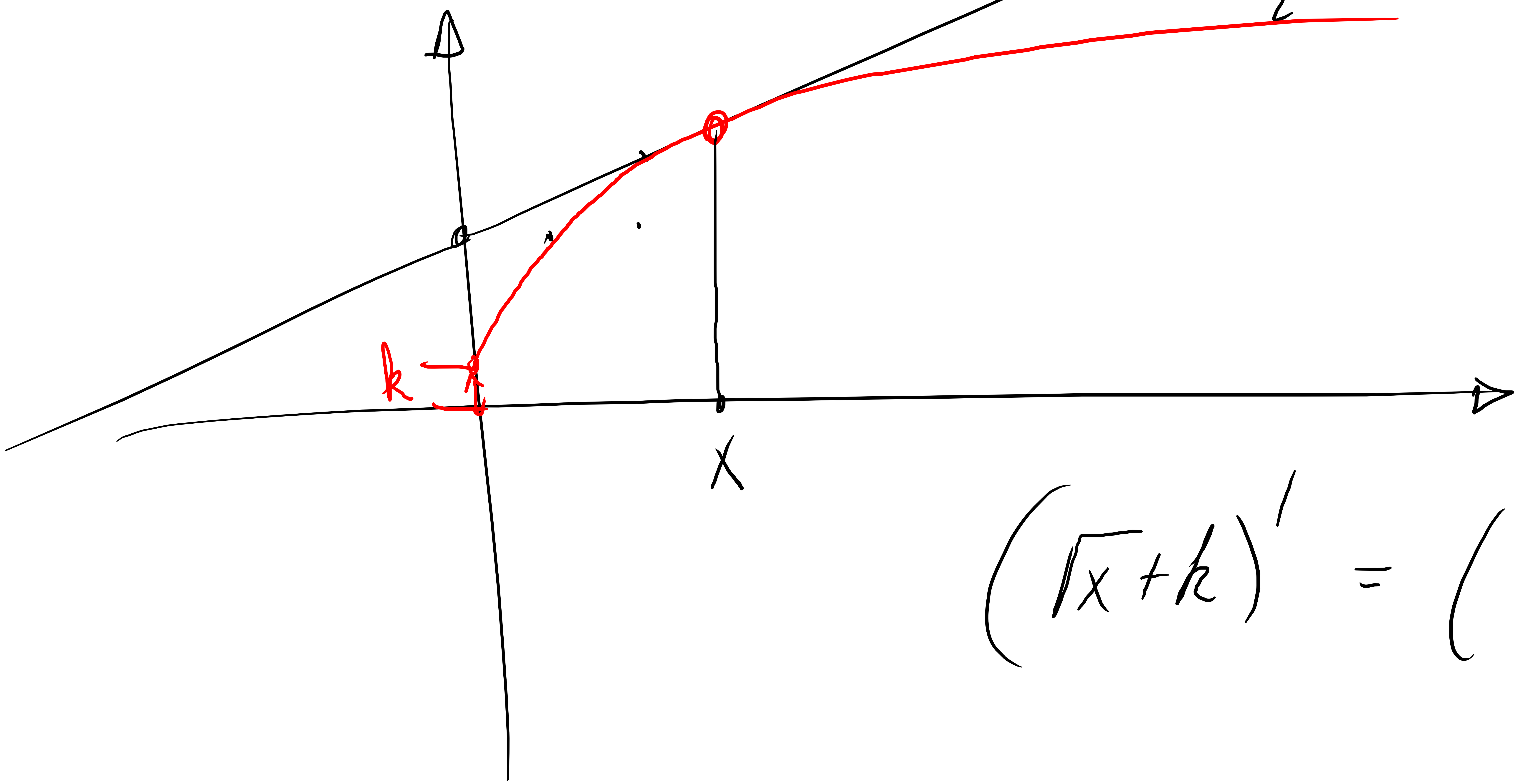
$x^2 - y^2 = -9$

$2xy = 0$

$2x^2 = 0 \Rightarrow x = 0$

$\Rightarrow y = \pm 3$

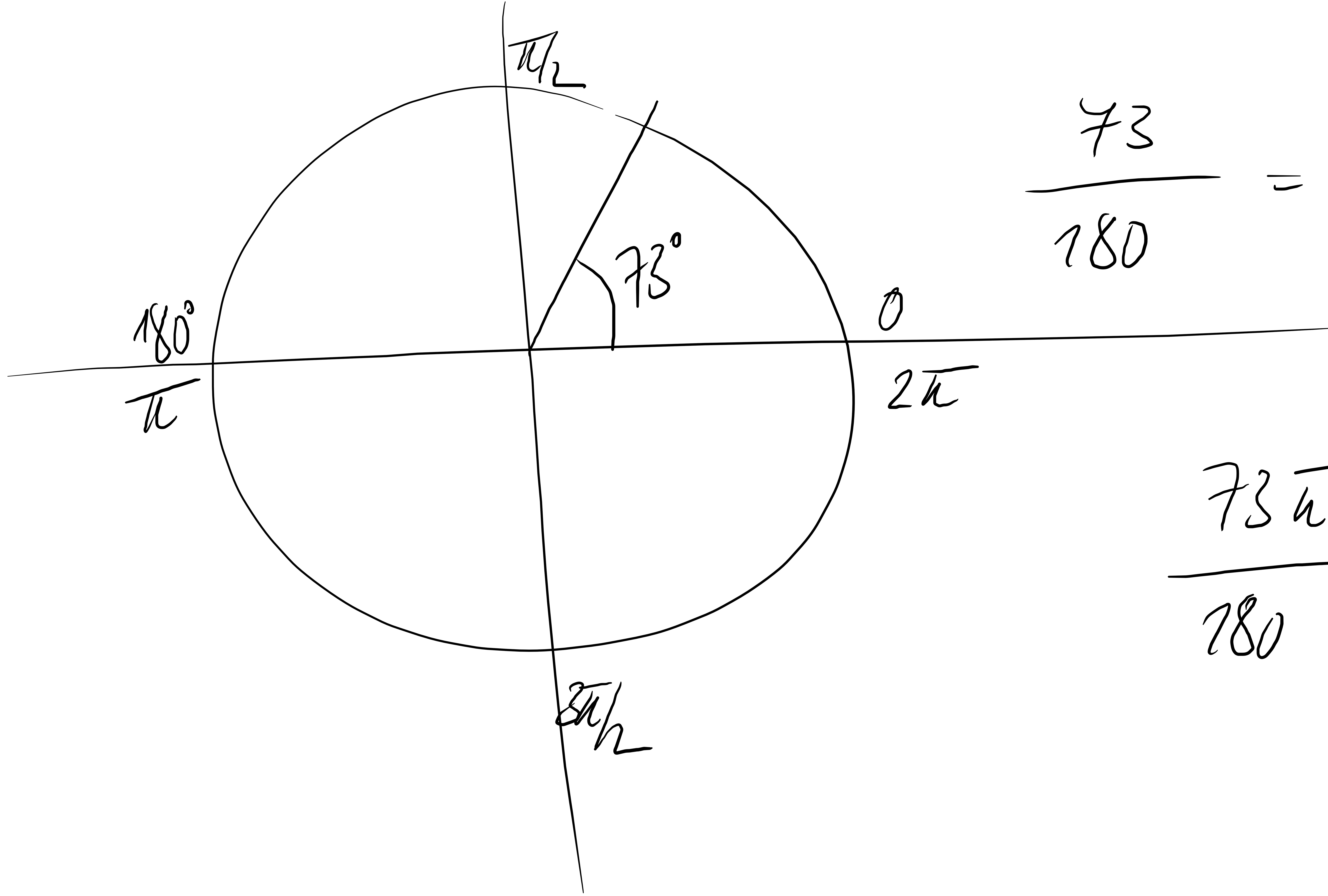
$$\sqrt{x} + k = \frac{x}{2} + 3$$



$$\left(\sqrt{x} + k\right)' = \left(\frac{x}{2} + 3\right)'$$

$$\left(\sqrt{x}\right)' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$



$$\frac{73}{180} = \frac{\alpha_{rad}}{\pi}$$

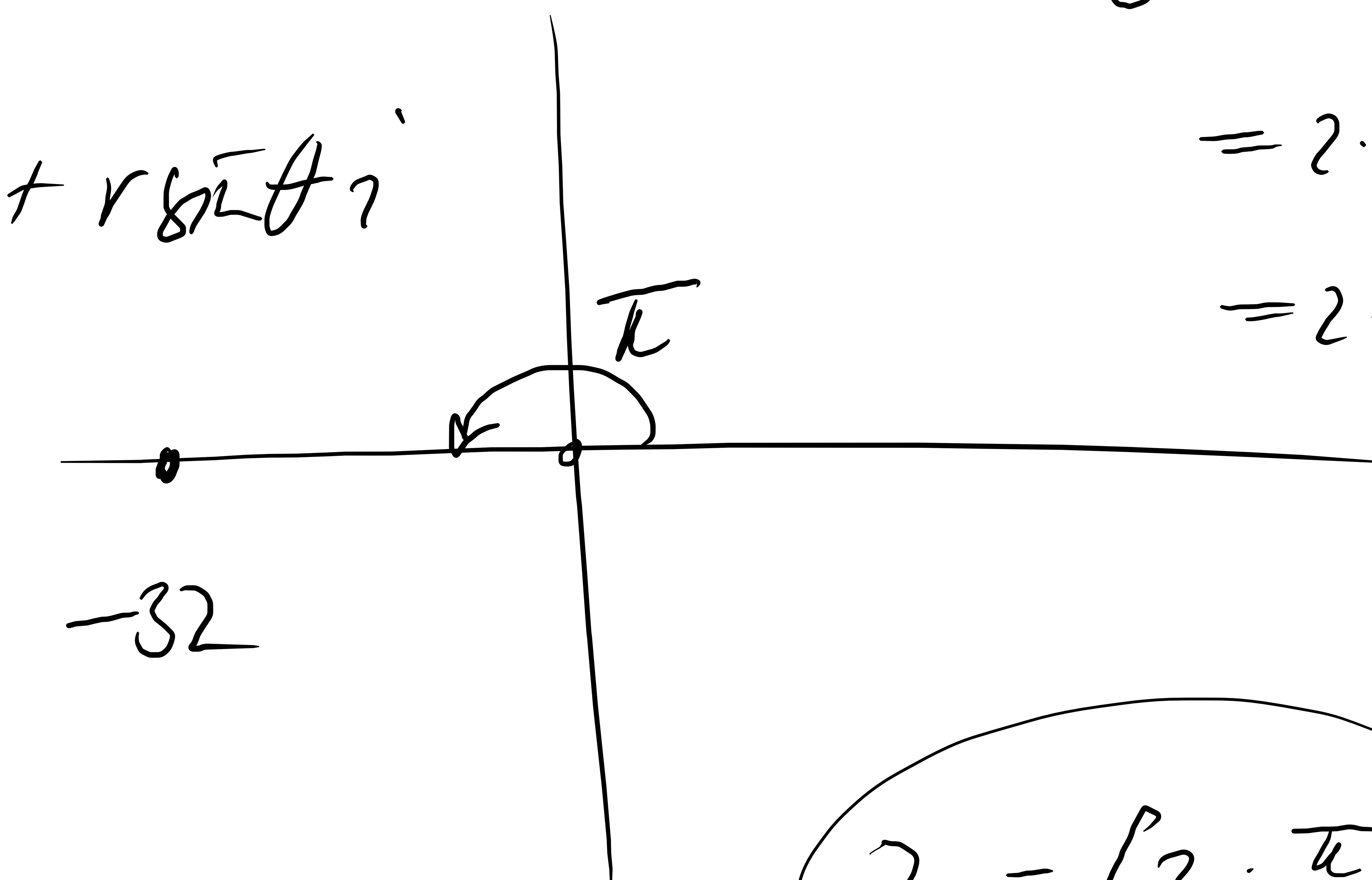
$$\frac{73\pi}{180} \simeq 0{,}405\,\pi$$

$$\simeq 1{,}2741 \text{ rad}$$

$$z^5 = -32 + 0i = [2^5; \pi]$$

$$32 = 2\cdot 16$$
$$= 2\cdot 2\cdot 8$$
$$= 2\cdot 2\cdot 2\cdot 2\cdot 2$$

$$z = [r; \theta] = r\cos\theta + r\sin\theta\, i$$



$$z^5 = [r^5; 5\theta] = [2^5; \pi]$$

$$\boxed{r=2} \qquad \sqrt[5]{32}$$

$$5\theta = \pi + k\cdot 2\pi$$

$$\theta = \frac{\pi}{5} + \frac{k2\pi}{5} \qquad k \in \mathbb{Z}_5$$

$$z_1 = \left[2; \frac{\pi}{5}\right] \qquad k=0$$

$$= 2\cos\frac{\pi}{5} + 2\sin\frac{\pi}{5}\, i$$

$$= \boxed{\frac{1+\sqrt{5}}{2} + \sqrt{\frac{5-\sqrt{5}}{2}}\cdot i}$$

$$z_2 = \left[2; \frac{3\pi}{5}\right]$$

$$\underset{a}{1}\, z^2 - \underbrace{3(i+1)}_{b} \cdot z + \underbrace{6+7i}_{c} = 0$$

$$\Delta = b^2 - 4ac$$

$$z_1 = \frac{-b+\sqrt{\Delta}}{2a} \qquad z_2 = \frac{-b-\sqrt{\Delta}}{2a}$$

$$\Delta = 9(i+1)^2 - 4 \cdot 1 \cdot (6+7i)$$

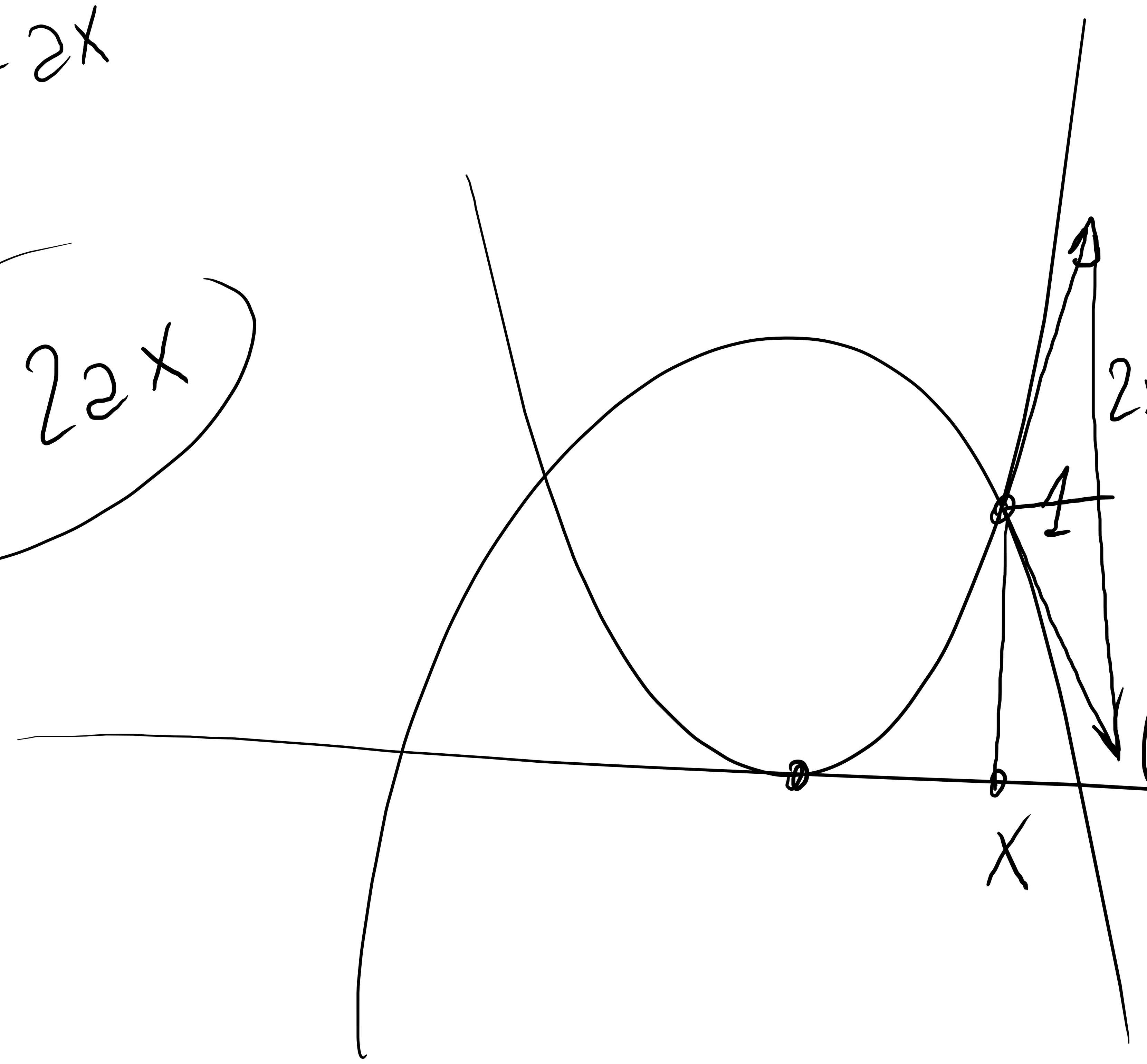
$$= 9(-1+2i+1) - 24 - 28i$$

$$= 18i - 24 - 28i$$

$$= \boxed{-24-10i}$$

$\frac{1}{2} - ax^2$

$-2ax$



$(x^2)' = 2x \qquad \begin{pmatrix} 1 \\ 2x \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{2}{\sqrt{2(1+a)}} \end{pmatrix}$

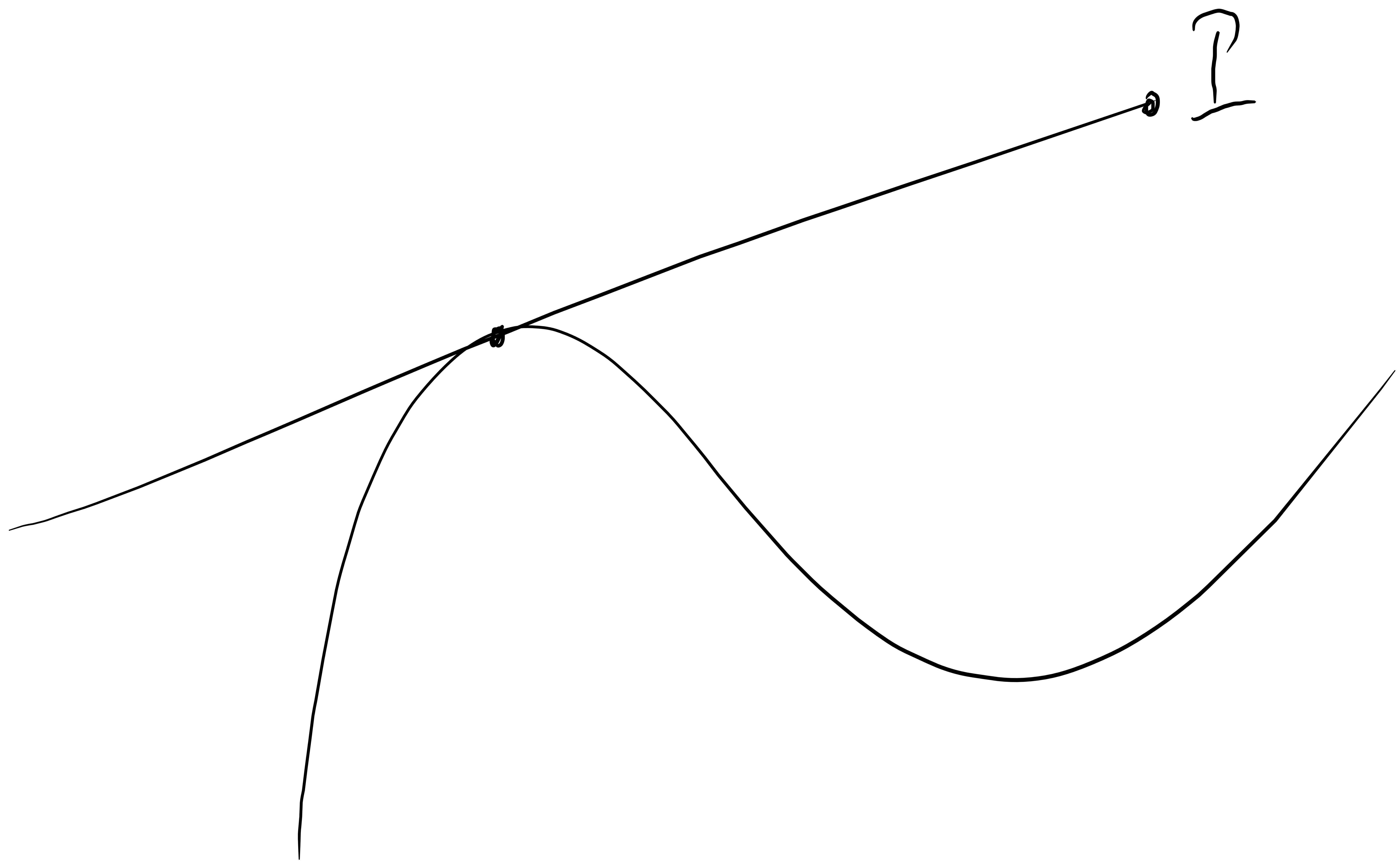
$\begin{pmatrix} 1 \\ -2ax \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-2a}{\sqrt{2(1+a)}} \end{pmatrix}$

$$\boxed{1 - \frac{4a}{2(1+a)} = 0}$$

$\frac{1}{2} - ax^2 = x^2$

$(1+a)x^2 = \frac{1}{2} \qquad x^2 = \frac{1}{2(1+a)} \qquad x = \frac{1}{\sqrt{2(1+a)}}$

$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = u_1 v_1 + u_2 v_2$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