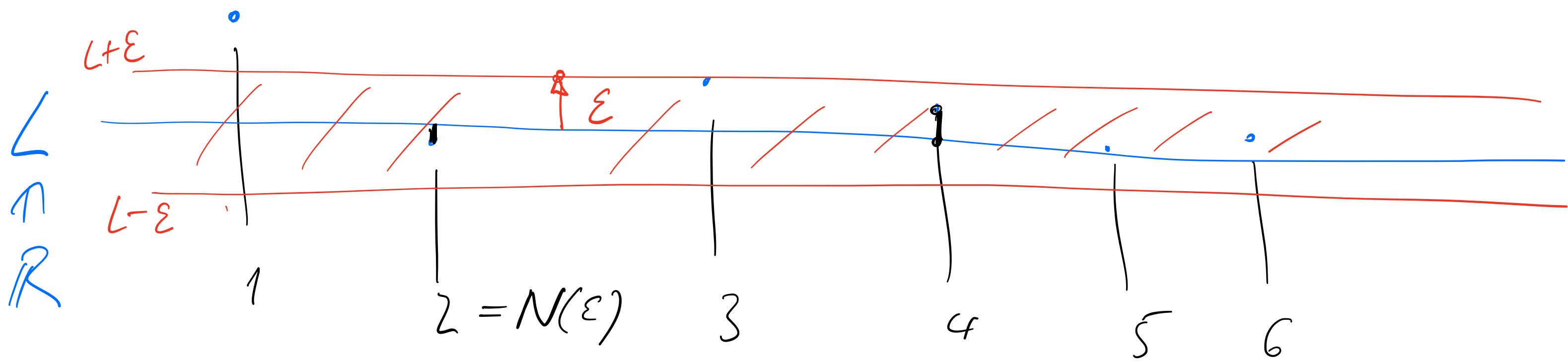
Soit x_n $n \in \mathbb{N}$ une suite.

pour tout

On dit que $\boxed{\lim_{n \to \infty} x_n = L}$ si $\forall \varepsilon > 0$

il existe $N(\varepsilon) \in \mathbb{N}$ tq. $|x_n - L| < \varepsilon$

$\exists$



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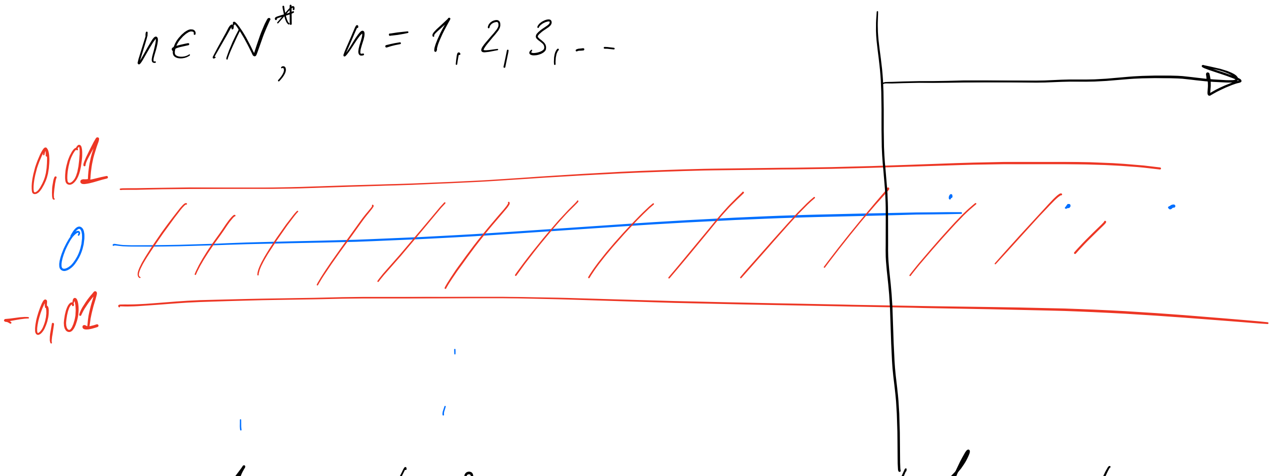
$x_n : \mathbb{N}^* \to \mathbb{R}$

$$x_n = \frac{1}{n} \quad n \geq 1 \qquad \lim x_n = 0$$

$n \in \mathbb{N}^*, \quad n = 1, 2, 3, \dots$



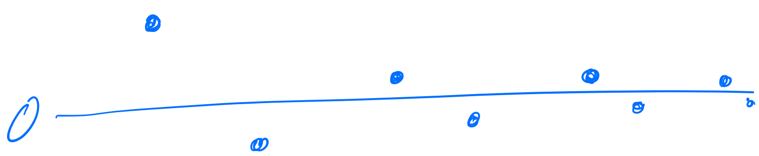
$n = 1$ $\qquad$ $n = 2$

$x_n = 1$

$$\left| \frac{1}{n} - 0 \right| < 0{,}01$$

$$N(\varepsilon) = 101 \qquad \frac{1}{0{,}01} = 100$$

$$y_n = (-1)^n \cdot \frac{1}{n}$$



Résultat:

$$\frac{1}{n} \xrightarrow{n \to +\infty} 0 \Leftrightarrow \lim \frac{1}{n} = 0$$

$$\Leftrightarrow \lim_{n \to +\infty} \frac{1}{n} = 0$$

preuve: Soit $\varepsilon > 0$.

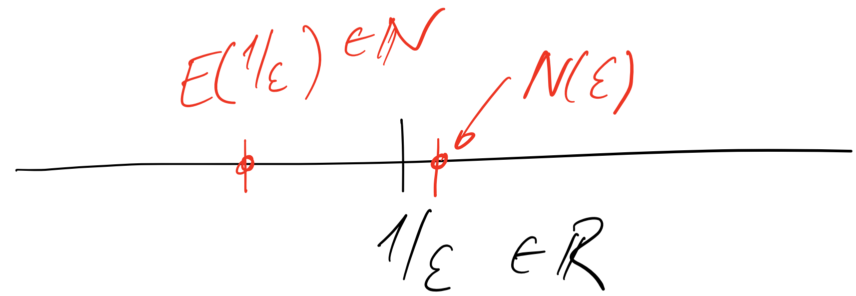
$$\left| \frac{1}{n} - 0 \right| \leftarrow |x_n - L|$$

$$\left|\frac{1}{n}-0\right| = \frac{1}{n} < \varepsilon \Leftrightarrow \frac{1}{\varepsilon} < n$$

partie entière

$$\Rightarrow N(\varepsilon) = \boxed{E\left(\frac{1}{\varepsilon}\right)+1}$$

$0,00234\overline{7}$

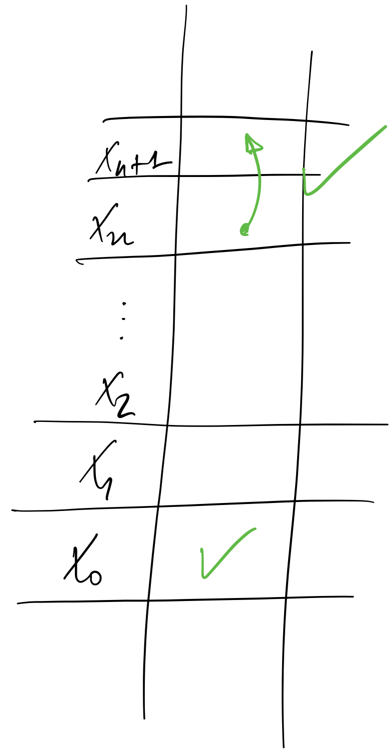


Suite définie par récurrence

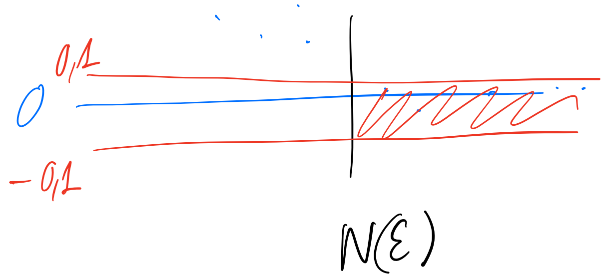
$$x_0 = C$$

« recette »

$$x_{n+1} = f(x_n)$$



$$\varepsilon = 0{,}1$$



$$\left| \frac{5}{n+2} - 0 \right| < 0{,}1$$

$\cdot\, n+2$ et $\div\, 0{,}1$

$$\frac{5}{n+2} < 0{,}1 \iff \frac{5}{0{,}1} < n+2$$

$$\iff 50 - 2 < n$$

$$\iff n > 48 \qquad N(\varepsilon) = 49$$

Soit x_n, y_n, z_n $n \in \mathbb{N}$ trois suites tq.

$$x_n \leq y_n \leq z_n \quad \forall n \in \mathbb{N}$$

$$\begin{array}{ccccc} x_n & \leq & y_n & \leq & z_n \\ \downarrow{\scriptstyle n \to +\infty} & & \downarrow{\scriptstyle n \to +\infty} & & \downarrow{\scriptstyle n \to +\infty} \\ L & & L & & L \end{array}$$

Si $\lim x_n = \lim z_n = L$, alors $\lim y_n = L$

$$\boxed{\text{Théorème des deux gendarmes}}$$

$$-1 \leq \cos(n) \leq 1$$

$$\boxed{\begin{array}{ccccc} \dfrac{-1}{n^2} & \leq & \dfrac{\cos(n)}{n^2} & \leq & \dfrac{1}{n^2} \\ \downarrow{\scriptstyle n \to +\infty} & & \downarrow{\scriptstyle n \to +\infty} & & \downarrow{\scriptstyle n \to +\infty} \\ 0 & & 0 & & 0 \end{array}}$$

$$\left|\frac{n}{n+1}-1\right| = \left|\frac{n}{n+1}-\frac{n+1}{n+1}\right|$$

$n \geqslant 0$

$$= \left|\frac{n-(n+1)}{n+1}\right| = \left|\frac{\overbrace{-1}^{<0}}{\underbrace{n+1}_{>0}}\right|^{<0}$$

$$|a| = \begin{cases} a & \text{si } a \geqslant 0 \\ -a & \text{sinon} \end{cases}$$

$$= -\frac{(-1)}{n+1}$$

$$= \frac{1}{n+1}$$

$$\mathbb{R}_+^* =]0; +\infty[$$

$$\frac{n!}{n^n} = \underbrace{\frac{n}{n}}_{\leq 1} \cdot \underbrace{\frac{(n-1)}{n}}_{<1} \underbrace{\frac{(n-2)}{n}}_{<1} \cdots \frac{2}{n} \cdot \underbrace{\frac{1}{n}}_{\leq 1}$$

$$\leq \frac{n!}{n^n} \leq 1$$

$n \geq 3$

$$\frac{3}{n-2} < 0{,}1 \overset{\cdot (n-2)}{\Leftrightarrow} 3 < 0{,}1 \cdot (n-2)$$

$$\overset{\div 0{,}1}{\Leftrightarrow} \frac{3}{0{,}1} < n-2$$

$$\frac{n!}{n^n} = \frac{1}{n} \cdot \frac{\overset{\leq 1}{n} \cdot \overset{\leq 1}{(n-1)} \cdot \overset{\leq 1}{(n-2)} \cdot \ldots \cdot \overset{\leq 1}{2 \cdot 1}}{n \cdot n \cdot n \cdots n}$$

$$\leq \frac{1}{n} \cdot 1 \leq \frac{1}{n}$$

$$\frac{n!}{n^n} \leq \frac{1}{n}$$

$$\frac{1}{n} \xrightarrow{n \to +\infty} 0$$

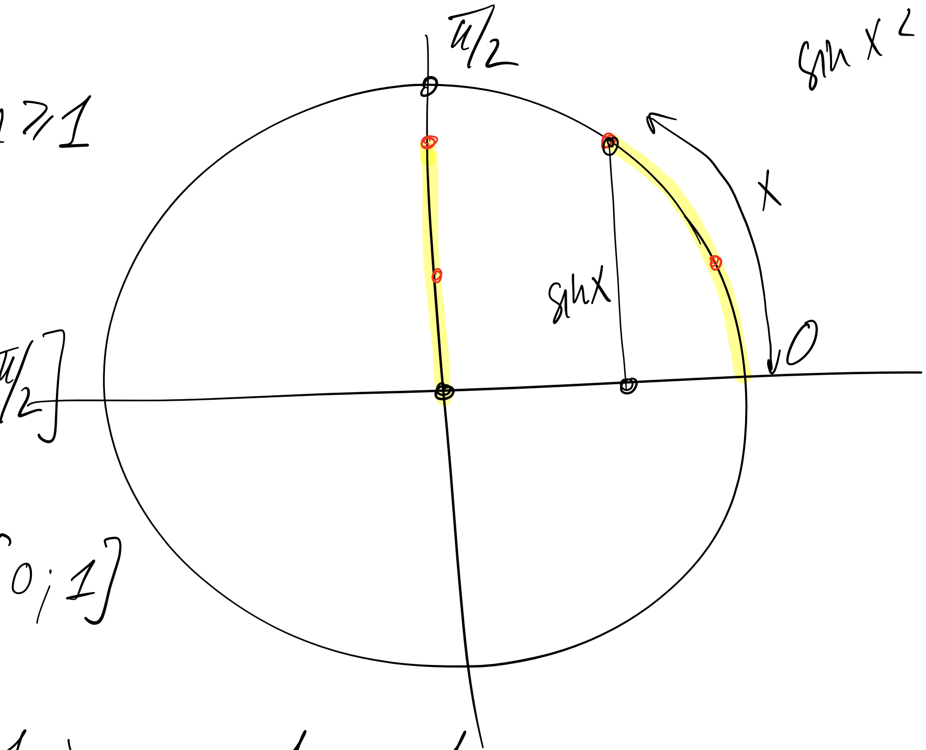
$$\frac{1}{n^k} \xrightarrow{n \to +\infty} 0 \qquad k \geq 1 \quad k \in \mathbb{N}$$

$\boxed{\text{Concrètement}}$

$$\lim \frac{1-n^3+n^5}{3n^5-n^2+2} \begin{matrix} \div n^5 \\ \\ \div n^5 \end{matrix} = \lim \frac{\overbrace{1/n^5}^{0} \overbrace{- 1/n^2}^{0} + 1}{3 \underbrace{- 1/n^3}_{0} + \underbrace{2/n^5}_{0}}$$

$$= 1/3$$

$n \sin\left(\frac{1}{n^2}\right) \qquad n \geq 1$



$\sin x < x$

$\frac{1}{n^2} > 0 \qquad \frac{1}{n^2} \in [0; \pi/2]$

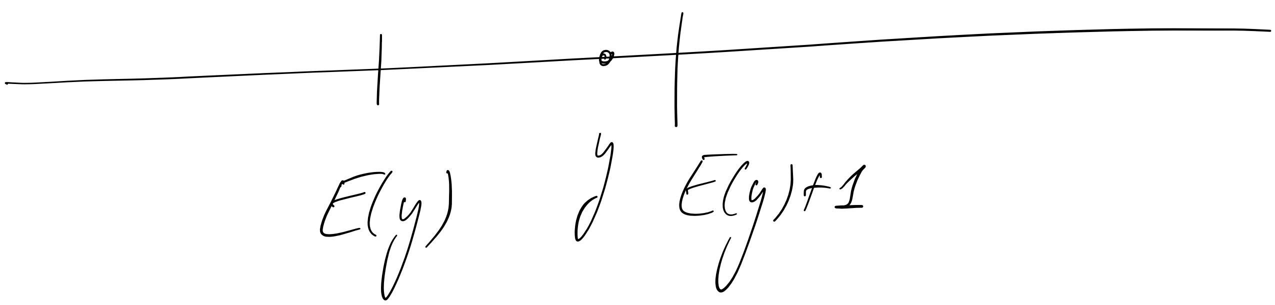
$\Rightarrow \sin\left(\frac{1}{n^2}\right) \in [0; 1]$

$0 \leq n \cdot \sin\left(\frac{1}{n^2}\right) \leq n \cdot \frac{1}{n^2} = \frac{1}{n}$

$$\sqrt{A} - B \cdot \boxed{\frac{\sqrt{A}+B}{\sqrt{A}+B}}_{\,1}$$

$$\sum_{i=1}^{n} i \stackrel{?}{=} \frac{n(n+1)}{2}$$

$$\begin{array}{cccccc} 1+ & 2+ & 3+ & \dots & + & n \\ n+ & (n-1)+ & & \dots & + & 1 \\ \hline n+1 & n+1 & & & & n+1 \end{array}$$



$$\boxed{E(y) \le y < E(y)+1}$$

Cas particulier :

$$E(nx) \leq nx < E(nx)+1$$

$$x_n = \boxed{\frac{1}{n} \cdot E(nx)}$$

$$y_n = \boxed{\frac{1}{n} \cdot nx} \qquad \Rightarrow \quad x_n \leq y_n \quad \forall n$$