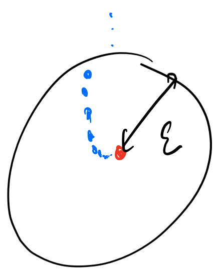
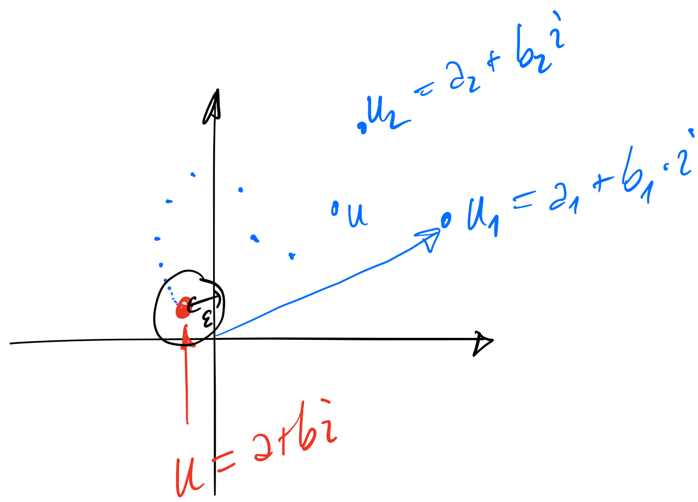
$\lim u_n = u \iff \forall \varepsilon > 0 \ \exists N_\varepsilon \in \mathbb{N}$ tq. $n \geq N_\varepsilon \Rightarrow |u_n - u| < \varepsilon$

$u_n \in \mathbb{C}$



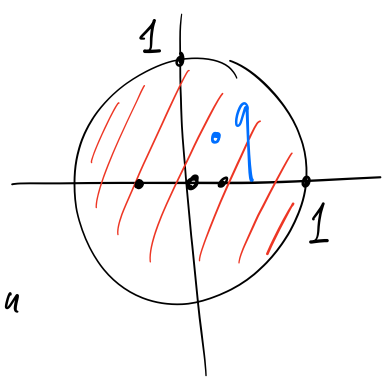


<u>Théorème:</u> Soit $q \in \mathbb{C}$ et $u_n = q^n \quad \forall n \in \mathbb{N}$

Si $|q| < 1$,

alors $\lim u_n = \lim q^n$

$= 0 \in \mathbb{C}$



$$\boxed{q^n \xrightarrow[n \to \infty]{} 0 \quad \text{si} \quad |q| < 1}$$ dans $\mathbb{R}$ et $\mathbb{C}$

Suite géométrique

Exemple:

$u_0 = 1 = q^0$ $\qquad v_0 = 2$

$u_1 = q$ $\qquad v_1 = 2q$

$u_2 = q^2$ $\qquad v_2 = 2q^2$

$u_3 = q^3$ $\qquad v_3 = 2q^3$

$\vdots \qquad \vdots$

Cas général :

$u_n = a \cdot r^n \qquad a, r \in \mathbb{R}$

Somme d'une suite géométrique

$u_n = r^n \qquad n \in \mathbb{N} \qquad r \in \mathbb{R}$

$$\sum_{n=0}^{k} u_n = u_0 + u_1 + u_2 + \dots + u_k$$

$$= r^0 + r^1 + \dots + r^k$$

$$= 1 + r + r^2 + \dots + r^k$$

Idée

$$(1+r+r^2+\dots+r^k)(1-r) = \overbrace{1+\cancel{r}+\cancel{r^2}+\dots+\cancel{r^{k-1}}+\cancel{r^k}}^{}$$
$$-\cancel{r}-\cancel{r^2}-\dots-\cancel{r^{k-1}}-\cancel{r^k}-r^{k+1}$$
$$= 1 - r^{k+1}$$

$$\Rightarrow \sum_{n=0}^{k} r^n \cdot (1-r) = 1 - r^{k+1}$$

$r \neq 1$

$$\Rightarrow \sum_{n=0}^{k} r^n = \frac{1 - r^{k+1}}{1-r}$$

si $|r| < 1$

$n \to \infty$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

$$\sum_{n=0}^{k} 2^n = \frac{1 - 2^{k+1}}{1-2} = 2^{k+1} - 1$$

2^6

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 256 - 1$$

<u>Exercice:</u> Démontrer que

$$\sum_{n=0}^{k} a r^n = a \cdot \frac{1 - r^{k+1}}{1-r} = a \cdot \frac{r^{k+1} - 1}{r - 1}$$

par récurrence sur k.

<u>preuve:</u> $\boxed{k = 0}$

$$\sum_{n=0}^{0} a r^n = a \cdot r^0 = a \overset{?}{=} a \cdot \frac{1 - r^{0+1}}{1-r} = a \cdot 1$$

✓

$\boxed{k \checkmark \Rightarrow k+1 \checkmark}$ hyp. de réc. $\displaystyle\sum_{n=0}^{k} a \cdot r^n = a \cdot \frac{1-r^{k+1}}{1-r}$

$$\sum_{n=0}^{k+1} a \cdot r^n = \sum_{n=0}^{k} a \cdot r^n + a \cdot r^{k+1}$$

$$\overset{\text{hyp. de réc.}}{=} a \cdot \frac{1-r^{k+1}}{1-r} + a \cdot r^{k+1}$$

$$= a \cdot \left(\frac{1-r^{k+1}}{1-r} + r^{k+1} \right)$$

$$= a \cdot \left(\frac{1-r^{k+1} + (1-r) \cdot r^{k+1}}{1-r} \right)$$

$$= a \cdot \left(\frac{1-r^{k+1} + r^{k+1} - r^{k+2}}{1-r} \right)$$

$$= a \cdot \frac{1-r^{(k+1)+1}}{1-r}$$

CQFD