Asymptotes

TE 1er décembre

$$f(x) = \frac{x+2}{x + \sqrt{x^2+1}}$$

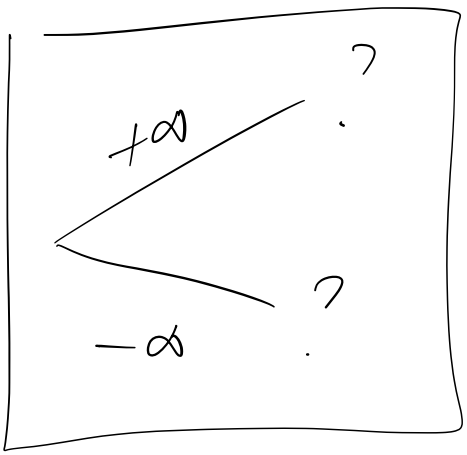
$D_f = \mathbb{R}$ $(x^2+1 > 0)$

A.V.

$x = -\sqrt{x^2+1}$?

$x^2 = x^2+1$

$0 = 1$ ↯



$$\lim_{x \to +\infty} f(x) = \ll \frac{+\infty}{+\infty + \infty} \gg = \ll \frac{\infty}{\infty} \gg \quad \text{Indéterminé}$$

$$\lim_{x \to -\infty} f(x) = \ll \frac{-\infty}{-\infty + \infty} \gg \quad \text{Indéterminé}$$

$$-x + e^x \longrightarrow +\infty$$

$-\infty \quad +\infty$

$$\frac{x+2}{x+\sqrt{x^2+1}} \cdot \frac{1/x}{1/x} \overset{x>0}{=} \frac{1+2/x}{1+\frac{1}{\sqrt{x^2}}\cdot\sqrt{x^2+1}} = \frac{1+2/x}{1+\sqrt{1+1/x^2}}$$

$\frac{1}{x}\sqrt{x^2+1} = \sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}$ $\qquad \frac{1}{x}$ $\qquad x \to +\infty$

$\frac{1}{2}$

A. H. à droite en $y = \frac{1}{2}$

$$\frac{x+2}{x+\sqrt{x^2+1}} \cdot \frac{1/x}{1/x} \overset{x<0}{=} \frac{1+2/x}{1+\frac{1}{x}\cdot\sqrt{x^2+1}} = \frac{1+2/x}{1-\frac{1}{\sqrt{x^2}}\sqrt{x^2+1}} = \frac{1+2/x}{1-\sqrt{1+1/x^2}}$$

$-\frac{1}{\sqrt{x^2}} = -\frac{1}{|x|} = -\frac{1}{-x} = \frac{1}{x}$ $\qquad x \to -\infty$

$x<0 \Rightarrow |x| = -x$

1 / 0

Rien à gauche.

$\frac{1+2/x}{1-\sqrt{1+1/x^2}}$ $\qquad x \to -\infty \to -\infty$ $\qquad$ Quel signe ?

$\hookrightarrow \frac{A}{B}$ avec $A>0$ et $B<0$ si $|x|>10$

$1+2/x > 0$ si $|x|>10$

$1/x^2 > 0 \Rightarrow 1+1/x^2 > 1 \Rightarrow \sqrt{1+1/x^2} > 1$

$\Rightarrow 1-\sqrt{1+1/x^2} < 0 \quad \forall x \in \mathbb{R}^*$

$$f(x) \sim \boxed{mx+h}$$

A.O. en $y = mx+h$

$$\frac{f(x)}{x} \sim m + \frac{h}{x} \sim m$$

$$f(x) - mx \sim h$$

$$m = \lim_{x\to\infty} \frac{f(x)}{x}$$

$$h = \lim_{x\to\infty} \left(f(x) - mx\right)$$

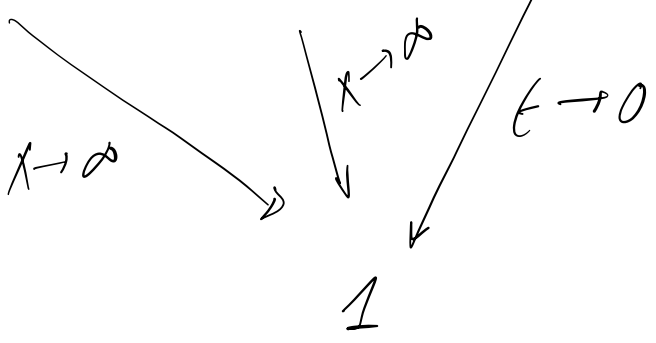
$$x \sin\left(\frac{1}{x}\right) \xrightarrow{x\to\infty} \ll \infty \cdot \sin(0) \gg = \ll \infty \cdot 0 \gg$$

Indéterminé.

$$\frac{\sin t}{t} \xrightarrow{t\to 0} 1$$

$t = \frac{1}{x}$ $\quad x\to\infty \Leftrightarrow t\to 0$

$$x \sin\frac{1}{x} = \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \frac{\sin t}{t}$$



$$\frac{x^2-4x}{x^2-4x+3} \xrightarrow{x\to\infty} \frac{x^2}{x^2} = 1$$

A.H. en $y=1$

$$x\sqrt{\frac{x}{x+1}} - x = x\left(\sqrt{\frac{x}{x+1}} - 1\right)\frac{\sqrt{\frac{x}{x+1}}+1}{\sqrt{\frac{x}{x+1}}+1}$$

$$= \frac{x\left(\frac{x}{x+1}-1\right)}{\sqrt{\frac{x}{x+1}}+1}$$

$$= x\cdot\frac{\frac{x-x-1}{x+1}}{\sqrt{\frac{x}{x+1}}+1}$$

$$= \frac{-x}{x+1}\cdot\frac{1}{\sqrt{\frac{x}{x+1}}+1}$$

$$\downarrow\ x\to\infty$$

$$-1\cdot\frac{1}{2} = -\frac{1}{2}$$