Valeur absolue

2.6.5

$x \in \mathbb{R}$

$$|x| = \sqrt{x^2} = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{sinon} \end{cases}$$

$$|5| = 5$$

$$|-10| = 10 = (-1)\cdot(-10) = -(-10) = 10$$

2.6.5 a) $\lim\limits_{x \to 0} \dfrac{|x|}{x} = « \dfrac{|0|}{0} » = « \dfrac{0}{0} »$ IND.

① $\lim\limits_{\substack{x \to 0 \\ >}} \dfrac{|x|}{x} = \lim\limits_{\substack{x \to 0 \\ >}} \dfrac{x}{x} = 1$

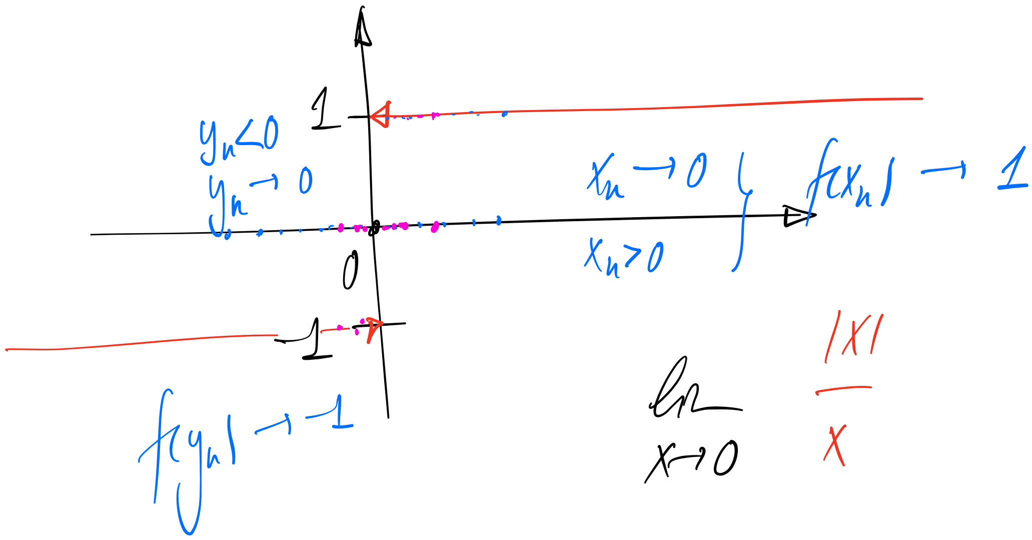
La limite à gauche est égale à 1.

si $x > 0$, $|x| = x$

② $\lim\limits_{\substack{x \to 0 \\ <}} \dfrac{|x|}{x} = \lim\limits_{\substack{x \to 0 \\ <}} \dfrac{-x}{x} = -1$

$x = -0,5$ $\dfrac{-(-0,5)}{-0,5}$

$$\Rightarrow \lim_{\substack{x \to 0 \\ >}} \frac{|x|}{x} \neq \lim_{\substack{x \to 0 \\ <}} \frac{|x|}{x} \Rightarrow \lim_{x \to 0} \frac{|x|}{x} \text{ n'existe pas.}$$



$$\lim_{x \to 0} \frac{|x|}{x}$$

<u>Théorème:</u> $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\lim_{x \to 0} \frac{\sin(2x)}{x} = \lim_{x \to 0} \left(\frac{\sin(2x)}{2x} \cdot \frac{2}{1} \right) = 2 \cdot \lim_{t \to 0} \frac{\sin t}{t}$$

Changement de variable: $t = 2x$; OK car $x \to 0$ ssi $t \to 0$

$\boxed{x > 0}$ de la droite

$$\frac{|x|}{x} = \frac{x}{x} = 1$$

$$|x| = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{sinon} \end{cases}$$

$\boxed{x < 0}$ de la gauche

$$\frac{|x|}{x} = \frac{-x}{x} = -1 \qquad (-x > 0)$$

Exemple: $x = -0{,}1$

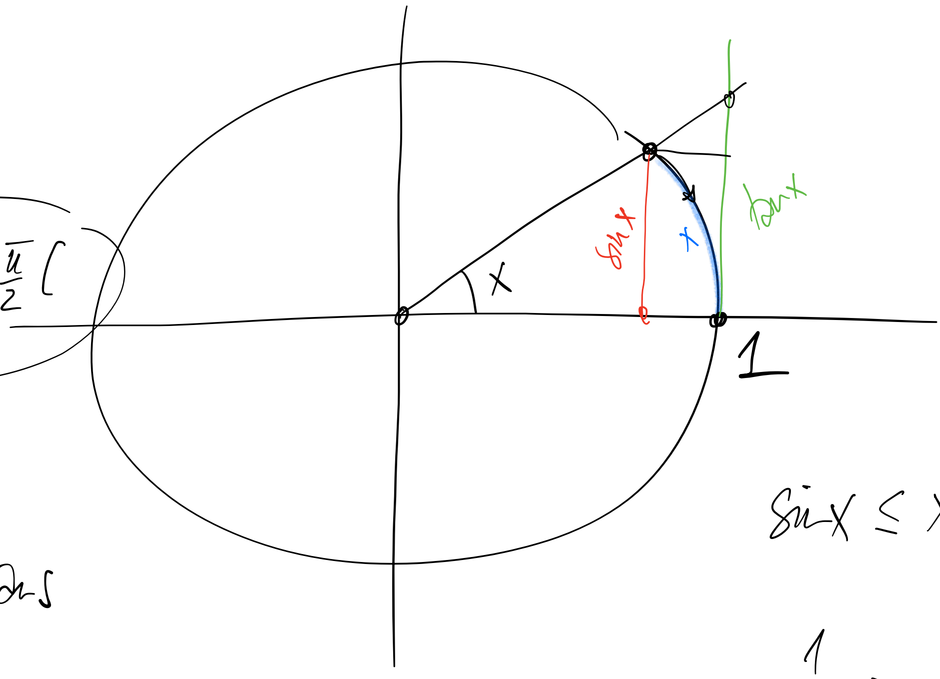
$$\frac{|x|}{x} = \frac{|-0{,}1|}{-0{,}1} = \frac{-(-0{,}1)}{-0{,}1}$$

Théorème: $\lim_{x\to 0} \frac{\sin x}{x} = 1$

preuve:

Soit $x \in]0; \frac{\pi}{2}[$

x est un angle en radians



$\sin x \leq x \leq \tan x$

$\frac{1}{\sin x} \geq \frac{1}{x} \geq \frac{1}{\tan x}$

$\frac{1}{\sin x} \geq \frac{1}{x} \geq \frac{\cos x}{\sin x}$ $\cdot \sin x$

$1 \geq \frac{\sin x}{x} \geq \cos x$

$x \to 0$: $1 \to 1$, $\frac{\sin x}{x} \to 1$, $\cos x \to 1$

$\lim_{x\to 0} \frac{\tan 7x}{\sin 3x}$

$\frac{\sin 7x}{\cos 7x} \cdot \frac{1}{\sin 3x} = \frac{\sin 7x}{7x} \cdot \frac{3x}{\sin 3x} \cdot \frac{7}{3} \cdot \frac{1}{\cos 7x}$