$\boxed{\text{Résoudre dans } \mathbb{C}}$

$$\frac{1}{2-i}z - 3 - \cancel{2i} = 1 - \cancel{2i} + (4-3i)z$$

$$\boxed{\frac{1}{2-i}}_{\in \mathbb{C}} z - \boxed{(4-3i)}_{\in \mathbb{C}} z = 4$$

conjugué

$$\frac{1}{2-i} \cdot \frac{2+i}{2+i} = \frac{2+i}{2^2+1^2}$$

$$(A-Bi)(A+Bi) = A^2+B^2$$

$$(2-i)(2+i) = 4 + \cancel{2i} - \cancel{2i} - (i)^2 = 4 - (-1) = 5$$

$$\left[\frac{2+i}{5} - (4-3i)\right] z = 4$$

$$\left[-\frac{18}{5} + \frac{16}{5}i\right] z = 4$$

$$(-18 + 16i) z = 20$$

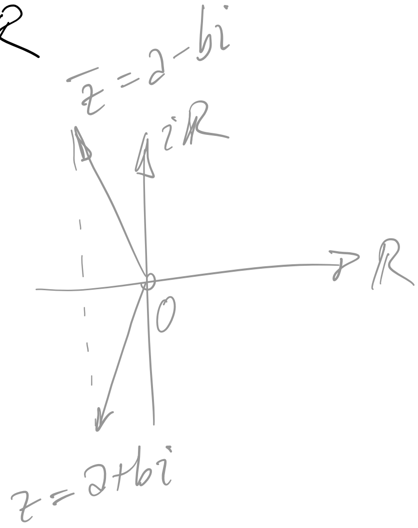
$$\boxed{(-9+8i)} z = 10 \qquad \cdot \frac{1}{-9+8i}$$

$$z = \frac{10}{-9+8i} \cdot \frac{-9-8i}{-9-8i} = \frac{-90-80i}{81+64} = -\frac{90}{145} - \frac{80}{145}i$$

$$= -\frac{18}{29} - \frac{16}{29}i$$

Conjugué et module

Déf: $z \in \mathbb{C}$, $z = a+bi$ $\quad a, b \in \mathbb{R}$



$\bar z$ se lit : le CONJUGUÉ de z

$$\boxed{\bar z = \overline{a+bi} = a - bi}$$

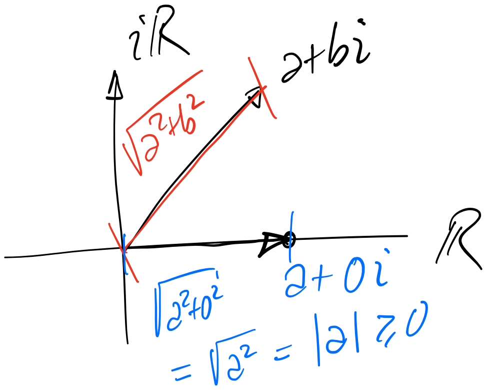
Remarque:

$$\bar z \cdot z = (a+bi)(a-bi) = a^2 - (bi)^2 = a^2 - b^2 i^2 = a^2 + b^2 = |z|^2$$

Déf: $|z| = \sqrt{a^2+b^2}$

$$\left(|z|^2 = a^2+b^2 = z \cdot \bar z \right)$$

$|z|$ se lit : le MODULE de z (norme)



$z \in \mathbb{R}$ et $\underbrace{z = a+bi}_{b=0}$

$$|z| = |a+0i| = \sqrt{a^2} = |a|$$

valeur absolue

Notation: $z = a+bi$

$\mathcal{I}(z) = b$ $\quad$ $\mathcal{R}(z) = a$

partie IMAGINAIRE de z

partie RÉELLE de z

Prop. $\overline{(z^n)} = \overline{z}^n$

preuve par réc. sur n :

$\overline{(z^0)} = \overline{1} = 1 = (\overline{z})^0$ $\boxed{n=0 \ \checkmark}$

$\boxed{n \checkmark \Rightarrow n+1 \checkmark}$ hyp. de réc. : $\overline{(z^n)} = \overline{z}^n$

$\overline{z}^{n+1} = \overline{z}^n \cdot \overline{z} = \overline{(z^n)} \cdot \overline{z} = \overline{z^n}$

car $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$ si $z, w \in \mathbb{C}$

En effet,

$\overline{(a+bi)} \cdot \overline{(c+di)} =$

$(a-bi)(c-di) = (ac-bd) - (ad+bc)i$

$= \overline{(ac-bd) + (ad+bc)i}$

$= \overline{z \cdot w}$

CQFD

$$a + bi = c + di$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$\Leftrightarrow \quad \begin{aligned} a &= c \\ b &= d \end{aligned}$$