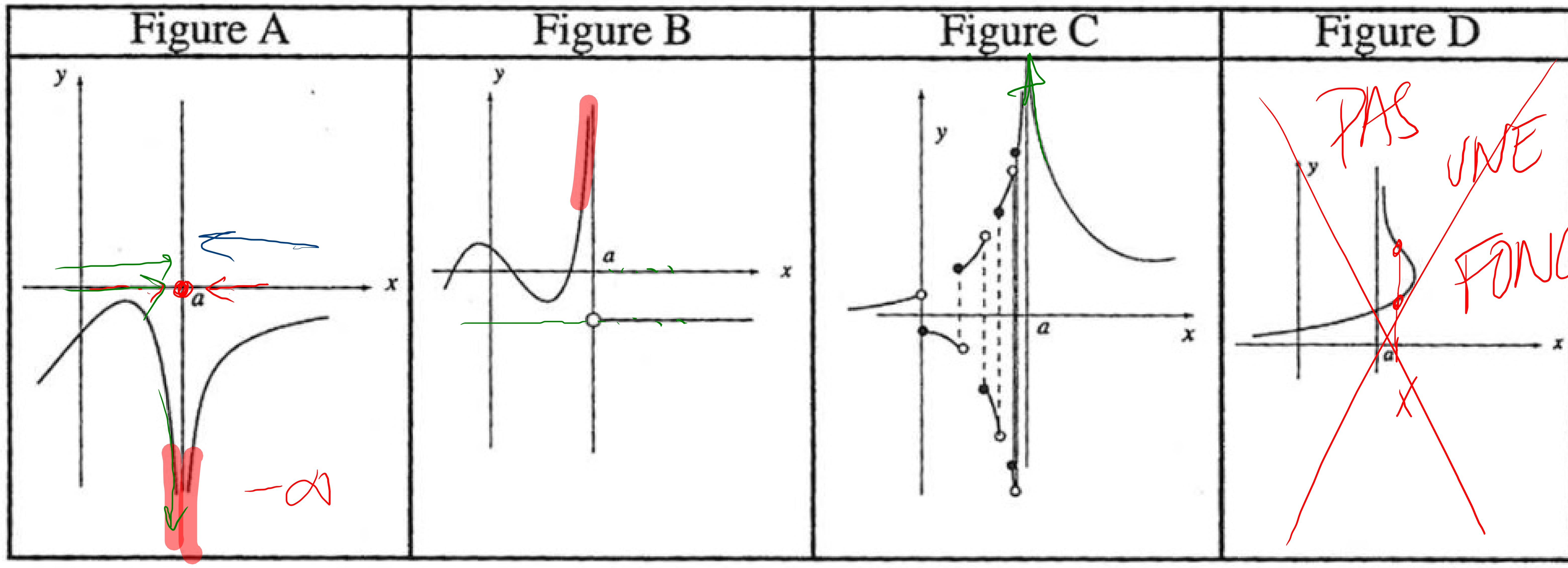
2.6.13 Dire pour chacune des quatre figures ci-dessous quelles sont les notations autorisées parmi 1), 2), …, 9) :

f(a) non définie

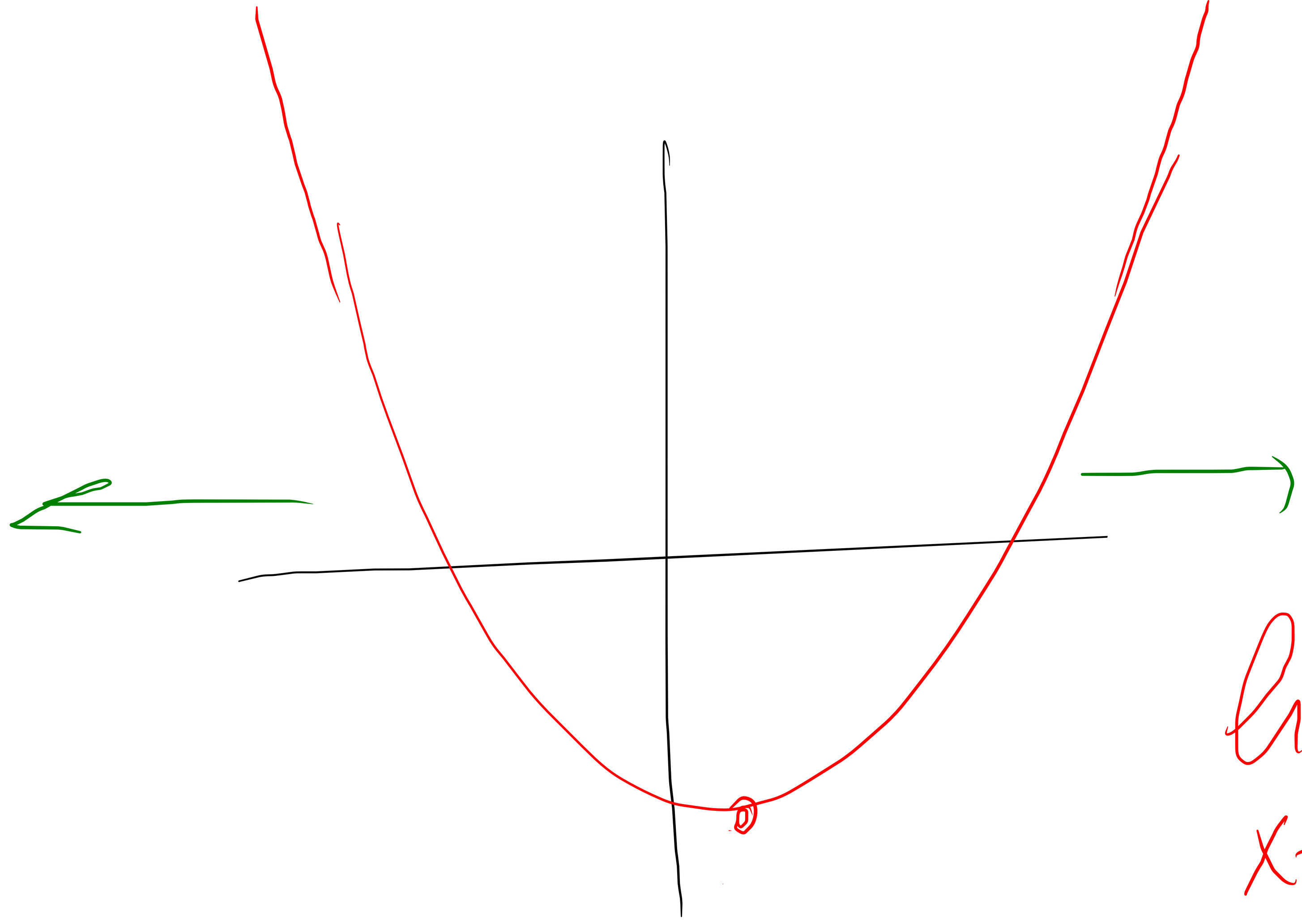


A: −∞ ; −∞ ; −∞

B: — ; +∞ ; —

$$\lim_{x\to a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases} \qquad \lim_{\substack{x\to a \\ <}} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases} \qquad \lim_{\substack{x\to a \\ >}} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$$

C: ∞ ; ∞ ; +∞



$ax^2+bx+c = f(x)$

$\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to\infty} x^3 = \infty$

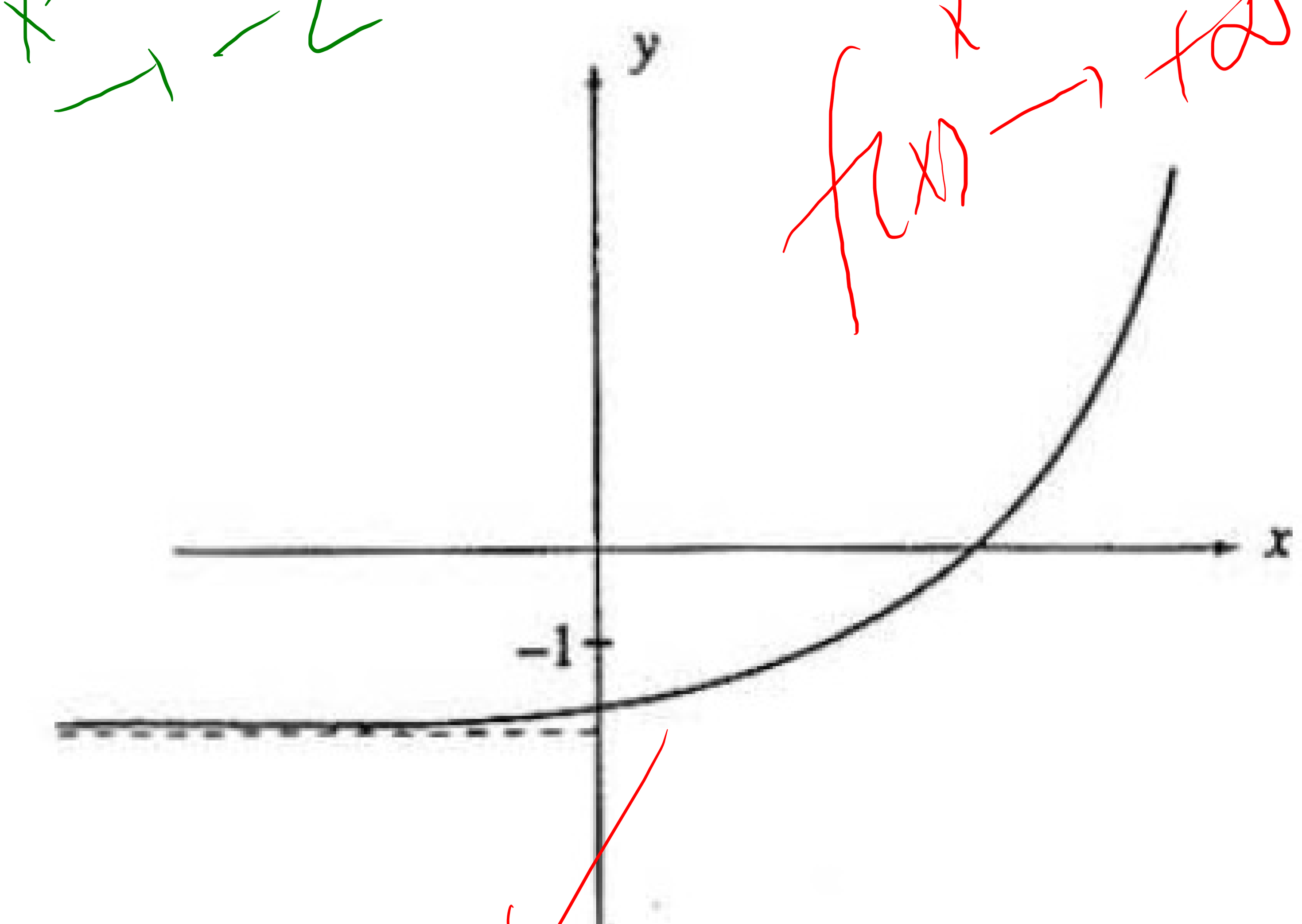
$\lim_{x\to-\infty} x^3 = -\infty$

2.6.15 Lire les limites : $\lim_{x\to-\infty} f(x)$, $\lim_{x\to+\infty} f(x)$ et $\lim_{x\to\infty} f(x)$.

a)

$f(x) \xrightarrow{x\to-\infty} -2$

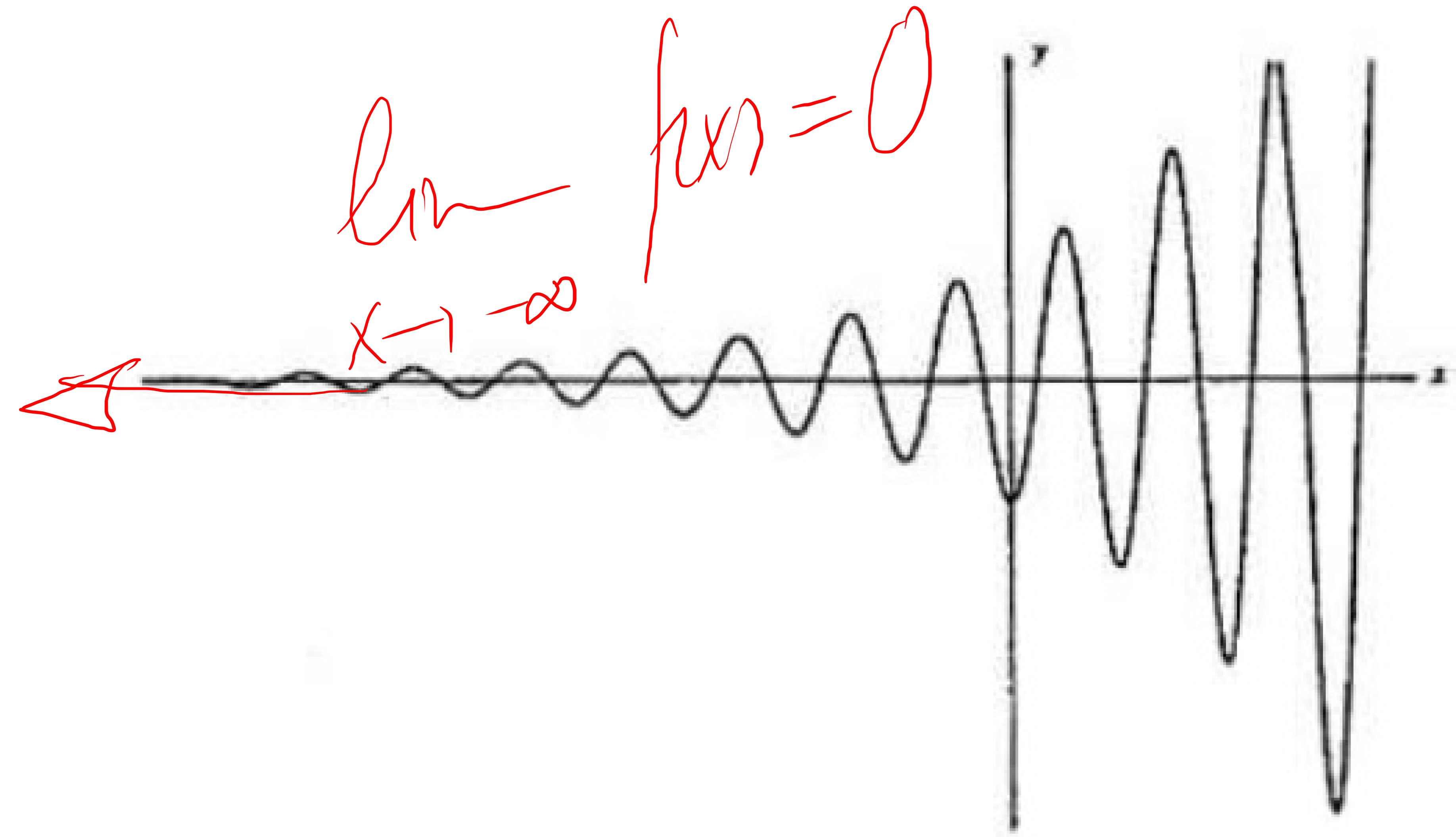
$f(x) \xrightarrow{x\to+\infty} +\infty$



$\cancel{\lim_{x\to\infty} f(x)}$

b)

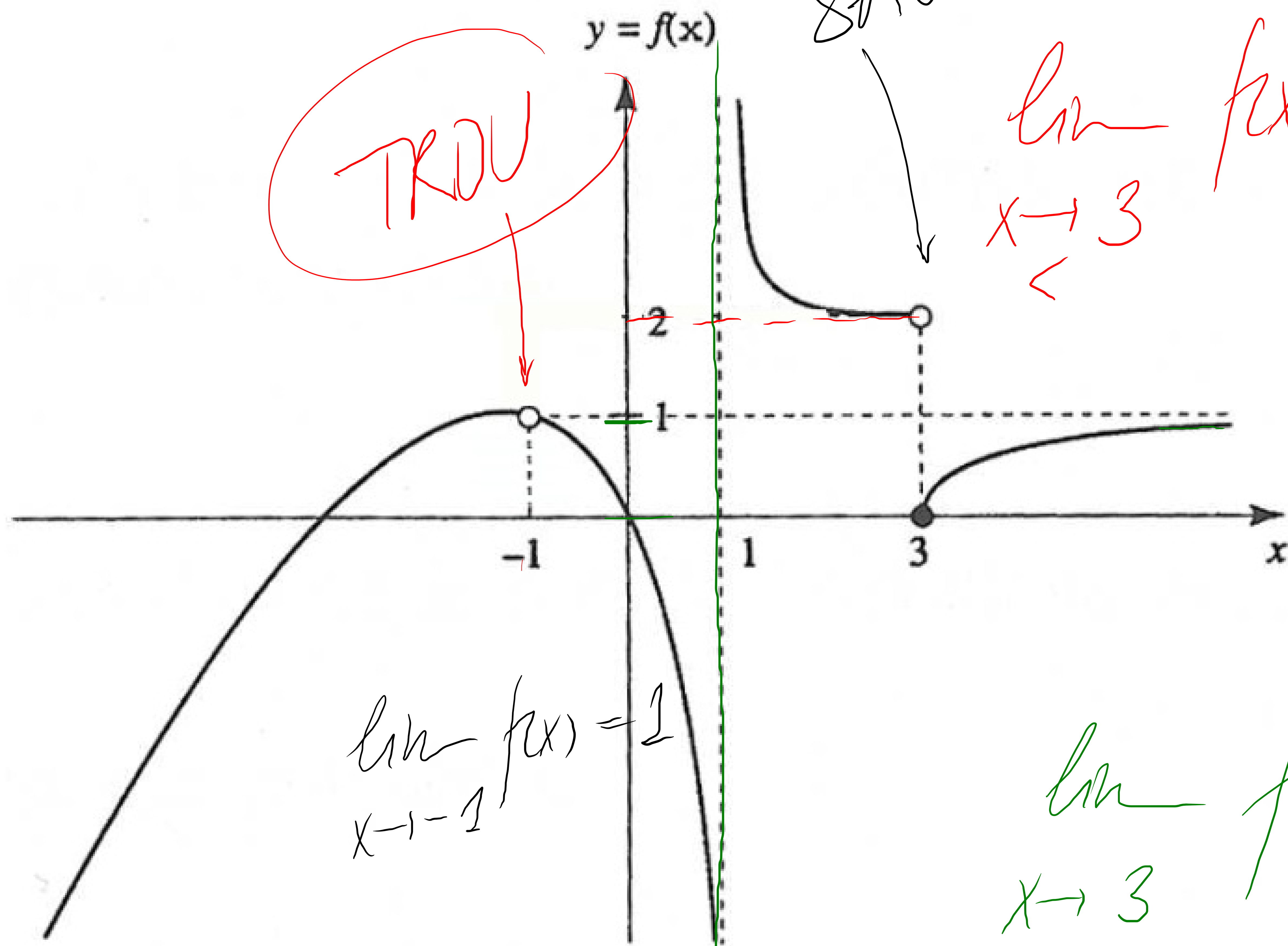
$\lim_{x\to-\infty} f(x) = 0$



$\lim_{x\to+\infty} f(x)$ pas défini

$x \sin x$

2.7.1 Déterminer la nature des discontinuités de la fonction f dont le graphe est représenté ci-dessous :



TROU

SAUT

$\lim_{\substack{x \to 3 \\ <}} f(x) = 2 \neq f(3)$

$\lim_{x \to -1} f(x) = 1$

$\lim_{\substack{x \to 3 \\ >}} f(x) = 0 = f(3)$

A.V.

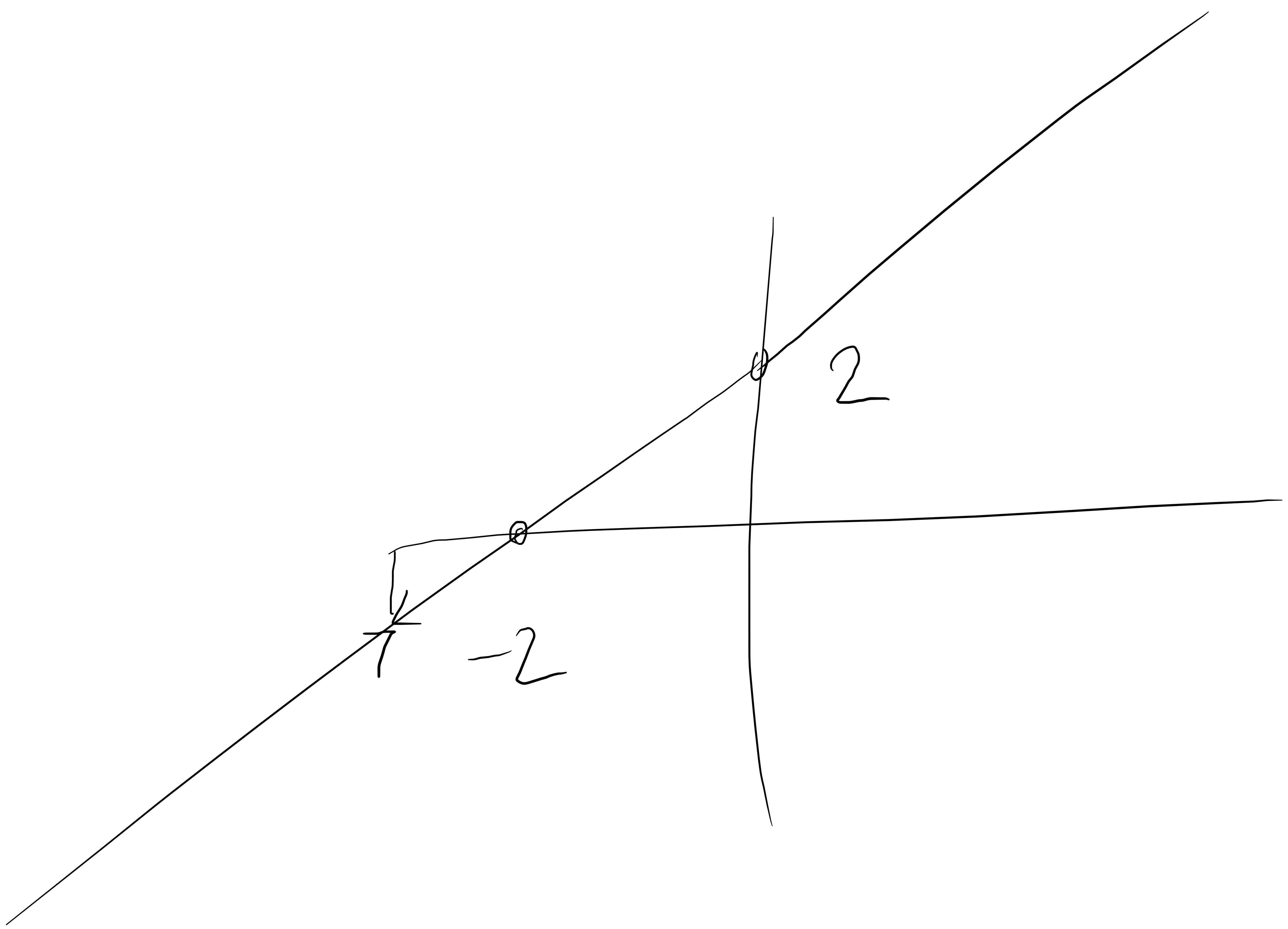
ASYMPTOTE VERTICALE

$$f(x) = \begin{cases} x = 1 & \text{si } x \in \mathbb{Q} \\ x = 0 & \text{sinon} \end{cases}$$

$$\frac{\cos x - 1}{x} \xrightarrow{x \to 0} \left\langle\!\left\langle \frac{1-1}{0} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle$$

IND

$$\frac{(x+2)(x-3)}{x-3} = \frac{x^2-x-6}{x-3}$$



$$\sqrt{x^2+x+1} - x \cdot \frac{\sqrt{x^2+x+1}+x}{\sqrt{x^2+x+1}+x} =$$

$$\frac{1-n}{n+2} \xrightarrow{n\to\infty} -1$$

<u>preuve.</u> Soit $\varepsilon > 0$

$$\left|\frac{1-n}{n+2} - (-1)\right| < \varepsilon$$

$$\Leftrightarrow \left|\frac{1-n}{n+2} + 1\right| < \varepsilon \Leftrightarrow \left|\frac{1-n}{n+2} + \frac{n+2}{n+2}\right| < \varepsilon$$

1

$$\Leftrightarrow \left|\frac{1-n+n+2}{n+2}\right| < \varepsilon \Leftrightarrow \left|\frac{3}{n+2}\right| < \varepsilon$$

$\frac{3}{n+2} > 0$ si $n \in \mathbb{N}$

$$\Leftrightarrow \frac{3}{n+2} < \varepsilon$$

$$\Leftrightarrow \frac{3}{\varepsilon} < n+2$$

$$\Leftrightarrow n > \frac{3}{\varepsilon} - 2$$

On pose donc, $N_\varepsilon = E\left(\frac{3}{\varepsilon} - 2\right) + 1$

Si $n \geq N_\varepsilon$ alors $\left|\frac{1-n}{n+2} - (-1)\right| < \varepsilon$ et donc $\lim \frac{1-n}{n+2} = -1$

$$\frac{x^2+4x+29}{x^2-2x+4} = \boxed{\frac{1+\frac{4}{x}+\frac{29}{x^2}}{1-\frac{2}{x}+\frac{4}{x^2}}} \xrightarrow{x\to\infty} \frac{1}{1} = 1$$

$$\lim_{x\to\infty} \frac{x^2+\boxed{4x+29}}{x^2\boxed{-2x+4}} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1$$

$$\frac{x^2+4x+29}{x^2-2x+4} \xrightarrow{x\to\infty} \frac{x^2}{x^2} = 1$$

$$\frac{x+1}{\sqrt{x^2+x+1}+x}\cdot\frac{1/x}{1/x} = \frac{1+1/x}{\frac{\sqrt{x^2+x+1}}{x}+1}$$

$$\xrightarrow{x\to+\infty}\ \frac{1+1/x}{\sqrt{1+1/x+1/x^2}+1}\ \xrightarrow{x\to+\infty}\ \frac{1}{1+1}=\frac{1}{2}$$

$$\xrightarrow{x\to-\infty}\ \frac{1+1/x}{-\sqrt{1+1/x+1/x^2}+1}\ \xrightarrow{x\to-\infty}\ «\frac{1}{0}» = \infty$$

$$0 = -1+1$$

$$\sqrt{x^2} = \begin{cases} x & \text{si } x \geq 0 \\ \boxed{-x \quad \text{si } x<0} \end{cases}$$

$$x = \sqrt{x^2} \quad \text{si } x>0$$

$$x = -\sqrt{x^2} \quad \text{si } x<0$$

$$\sqrt{x^2+x+1}-x \cdot \frac{\sqrt{x^2+x+1}+x}{\sqrt{x^2+x+1}+x} =$$

$$\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x} = \frac{x+1}{\sqrt{x^2+x+1}+x}$$