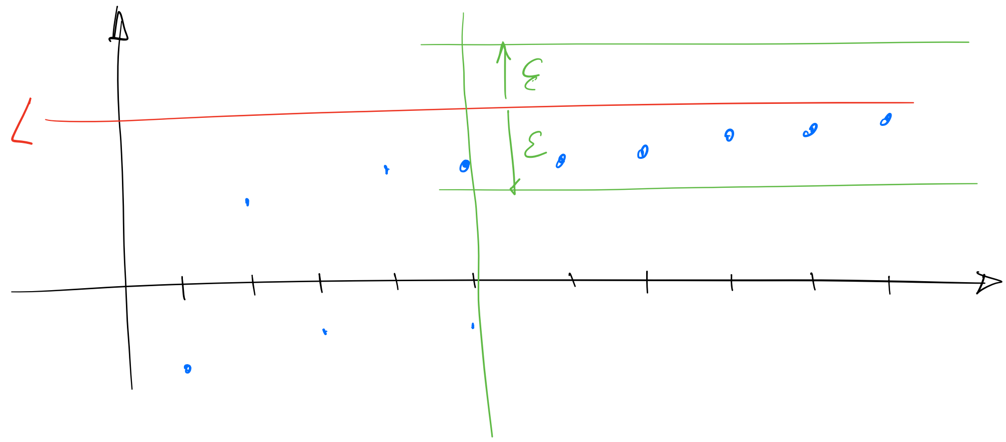
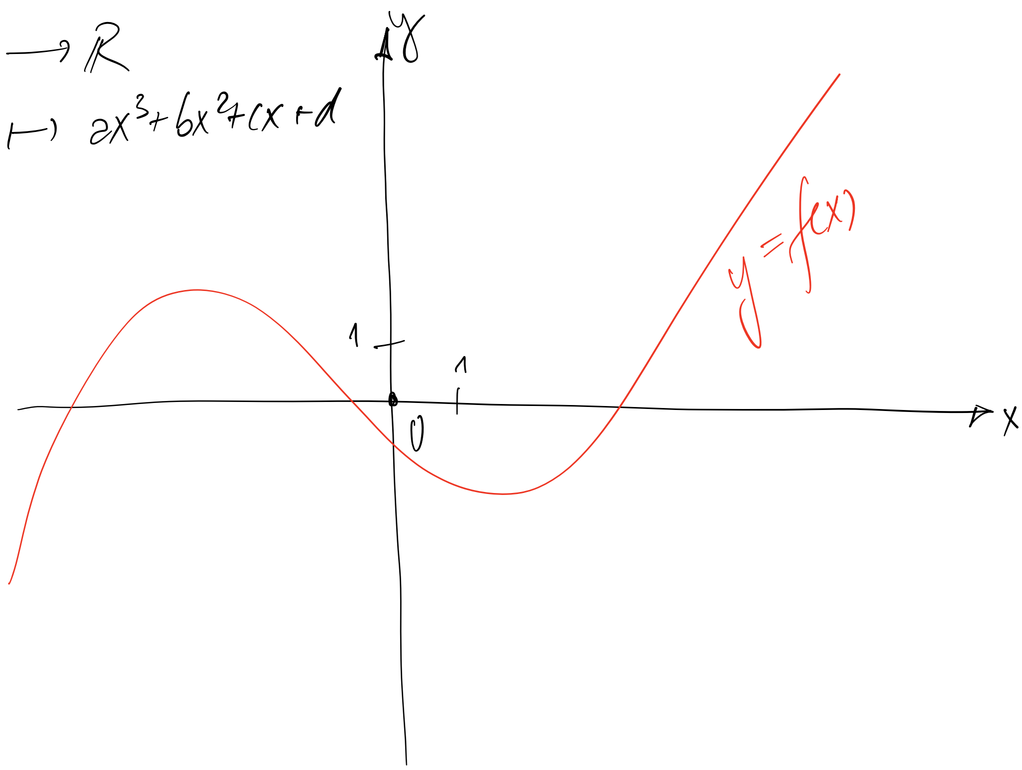


$f: \mathbb{R} \to \mathbb{R}$

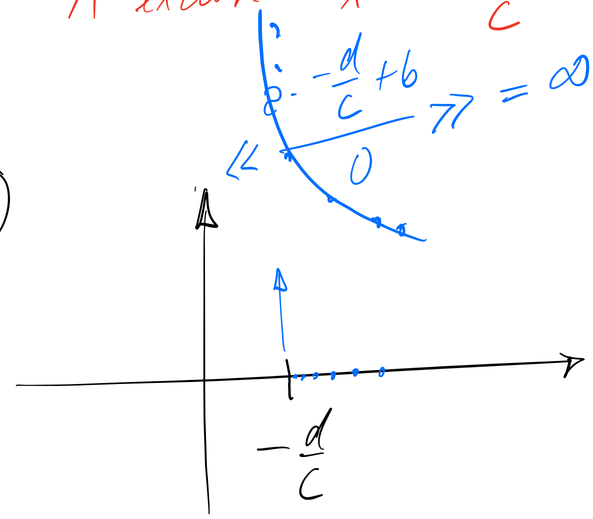
$x \mapsto ax^3 + bx^2 + cx + d$



$$f : \mathbb{R} - \left\{ -\frac{d}{c} \right\} \to \mathbb{R}$$
$$x \mapsto \frac{ax+b}{cx+d}$$

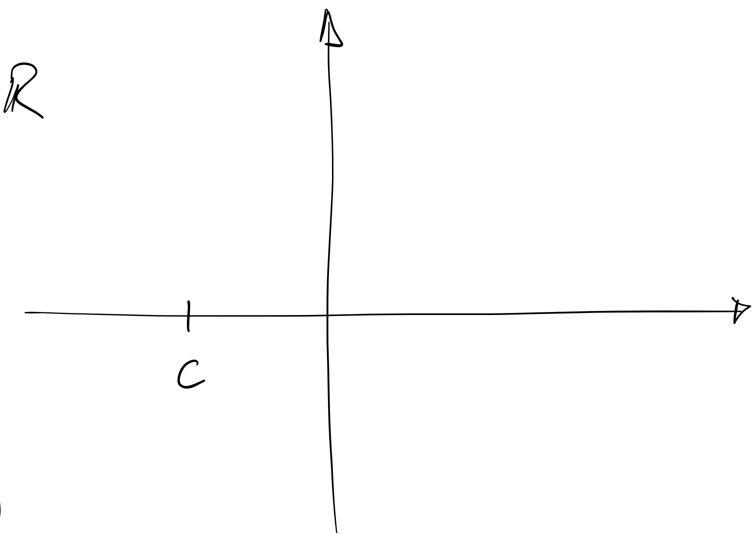
A' exclure : $x = -\frac{d}{c}$

$c \neq 0$

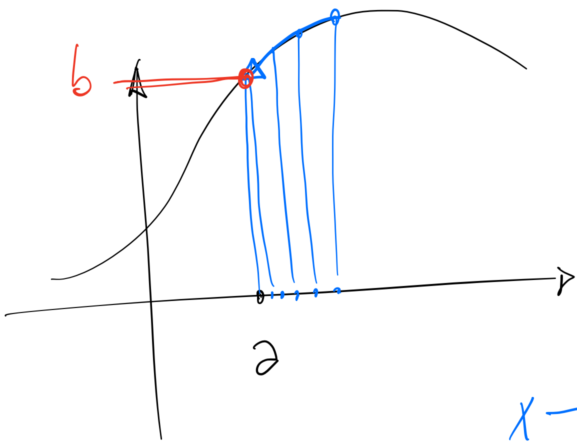


$$\boxed{\lim_{x \to -\frac{d}{c}} f(x) = \infty}$$

$$f : \mathbb{R} \to \mathbb{R}$$



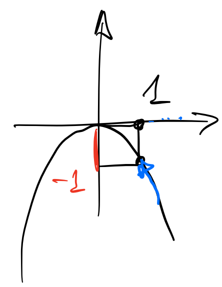
$$\lim_{x \to c} f(x)$$



$x \to a$

$f(x) \to b$

$$\lim_{x \to 1} -x^2 = -1$$

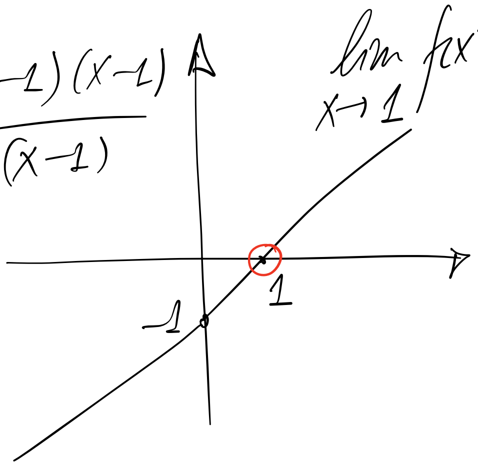


$\mathbb{R} - \{1\} \to \mathbb{R}$

$$f(x) = \frac{x^2 - 2x + 1}{x - 1} = \frac{(x-1)(x-1)}{(x-1)} = (x-1)$$

$x \neq 1$

f n'est pas déf. en $x=1$



$$\lim_{x \to 1} f(x) = 0$$

$$= \text{«} \frac{1^2 - 2 + 1}{1 - 1} \text{»} = \text{«} \frac{0}{0} \text{»}$$

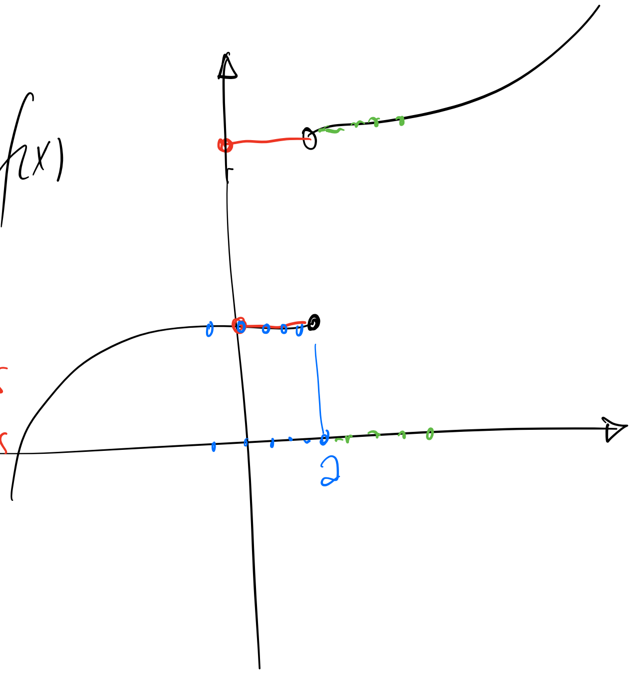
Indétermination

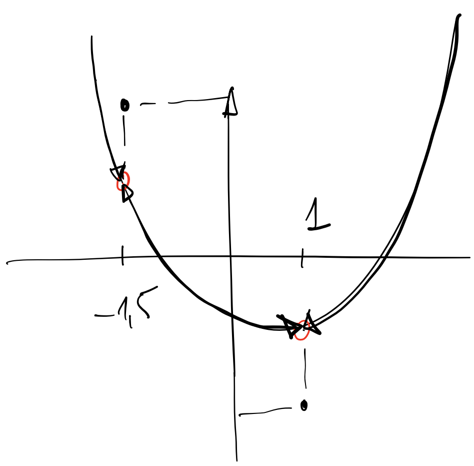
$$g(x) = \frac{x^2 - 2x + 1}{x - 2}$$

$$\lim_{x \to 2} g(x) = \text{«} \frac{1}{0} \text{»} = \infty$$

$\lim\limits_{\substack{x \to a \\ <}} f(x)$

par valeurs inférieures





$$f(x) = \begin{cases} 2 & \text{si } x = -1,5 \\ -2 & \text{si } x = 1 \\ (x+1)(x-3) & \text{sinon} \end{cases}$$

$\boxed{2.6.1}$ a)

$$\left.\begin{array}{l} \lim\limits_{\substack{x \to a \\ >}} f(x) = 6 \\ \lim\limits_{\substack{x \to a \\ <}} f(x) = 6 \end{array}\right\} \Rightarrow \lim\limits_{x \to a} f(x) = 6$$

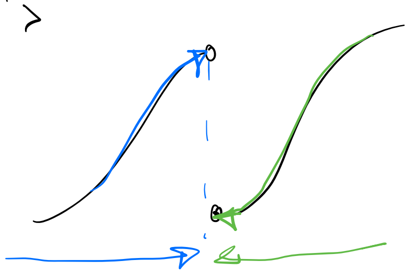
b) $\lim\limits_{\substack{x \to a \\ <}} f(x) = \lim\limits_{\substack{x \to a \\ >}} f(x) = 6 \Rightarrow \lim\limits_{x \to a} f(x) = 6 \; (\neq f(a))$

c) $\lim\limits_{x \to a^<} f(x) = b$ valeurs de x inferieures à celles de a

$\lim\limits_{x \to a^>} f(x) = f(a)$

Vu que $b \neq f(a)$,

$\lim\limits_{x \to a} f(x)$ n'existe pas



Calculer une limite : $\lim\limits_{x \to a} f(x)$

① Remplacer x par a dans $f(x)$ (ex. 2.6.1)

② Si cela donne une

INDÉTERMINATION, • par factorisation

• par la méthode du conjugué

lever cette indétermination.

2.6.2

a) 2	e) 2
b) 14	f) $4/3$
c) 0	g) -1
d) -5	h) $1/2$

2.6.3

nombre ($\pm\infty$)

IND.

a) $\lim\limits_{x \to 3} \frac{x-3}{2x-6} = « \frac{3-3}{2\cdot 3-6} » = « \frac{0}{0} »$

$= \lim\limits_{x \to 3} \frac{\cancel{(x-3)}\,1}{2\cancel{(x-3)}} = \frac{1}{2}$

$$\boxed{\frac{x-3}{2x-6} \underset{\downarrow}{=} \frac{x-3}{2(x-3)} = \frac{1}{2}} \xrightarrow{x\to 3} \boxed{\frac{1}{2}}$$

$$«\frac{3-3}{2\cdot 3-6}» = «\frac{0}{0}» \quad \text{INDÉTERMINÉ}$$

fractions de polynômes

nombres

b) $$\frac{100x^2}{x} = \frac{100x\cdot \cancel{x}}{\cancel{x}} = 100x \xrightarrow{x\to 0} 0$$

$«\frac{0}{0}»$ IND.

$$\Rightarrow \boxed{\lim_{x\to 0} \frac{100x^2}{x} = 0}$$

2.6.4

$$«\frac{16-16}{\sqrt{16}-4}» = «\frac{0}{0}» \quad \text{IND.}$$

a) $$\frac{x-16}{\sqrt{x}-4} = \frac{(x-16)\cdot(\sqrt{x}+4)}{(\sqrt{x}-4)\cdot(\sqrt{x}+4)} = \frac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{(x-16)}\cdot 1} = \sqrt{x}+4$$

$$(A-B)(A+B) = A^2-B^2$$

$$\sqrt{x}+4 \xrightarrow{x\to 16} 8 = \sqrt{16}+4$$

$$\frac{x-9}{\sqrt{x}-3} \xrightarrow{x\to 9} ?$$

$$\lim_{x\to 16} \frac{x-16}{\sqrt{x}-4} = 8$$

$$«\frac{0}{0}»$$

$$\boxed{\frac{x-9}{\sqrt{x}-3} = \frac{(x-9)(\sqrt{x}+3)}{(x-9)} \xrightarrow{x\to 9} 6}$$

$$\Rightarrow \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} = 6$$

→ Vendredi 8 septembre :

$\boxed{2.6.3 \text{ à } 2.6.5}$

→ Mercredi 13 septembre :

2.5.1 à 2.5.9 ← dernier délai

2.5.11 / $\underbrace{2.5.12}$

x_n est géométrique si $x_n = a \cdot q^n$

$a, q \in \mathbb{R}$

2.5.9

$u_1 = 2$

$u_{n+1} = \dfrac{u_n}{u_n + 1}$

$u_2 = \dfrac{2}{2+1} = \dfrac{2}{3}$

$u_3 = \dfrac{2/3}{2/3+1} = \dfrac{2}{3} \cdot \dfrac{3}{5} = \dfrac{2}{5}$

(1) $u_n > 0 \quad \forall n \geq 1$

$\boxed{n=1}$ $u_1 = 2 > 0$ ✓

$\boxed{n ✓ \Rightarrow n+1 ✓}$ $u_n > 0 \Rightarrow u_n + 1 > 0 \Rightarrow \dfrac{1}{u_n + 1} > 0$

$$\Rightarrow \frac{u_n}{u_n+1} > 0 \Rightarrow u_{n+1} > 0 \quad \text{CQFD}$$

② $u_{n+1} = \dfrac{u_n}{u_n+1} < u_n \Rightarrow u_n$ est strict. décr.

> 1 car $u_n > 0$