

$$x \in \mathbb{R}$$

$$|x| = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{si } x < 0 \end{cases}$$

$$|1-x^2| = 1 + |x|$$

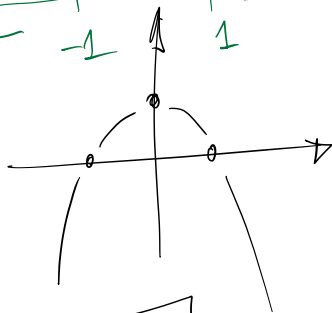
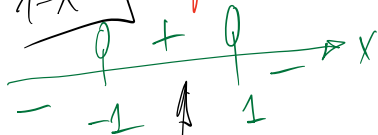
$$1-x^2 = 1+x$$

$$x^2+x=0$$

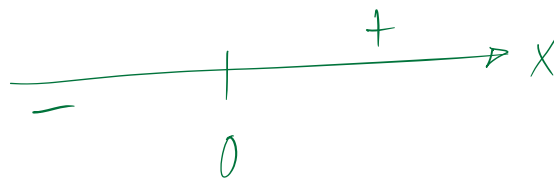
$$x(x+1)=0$$

$$x=0 \text{ / } x=-1$$

Signe $1-x^2$



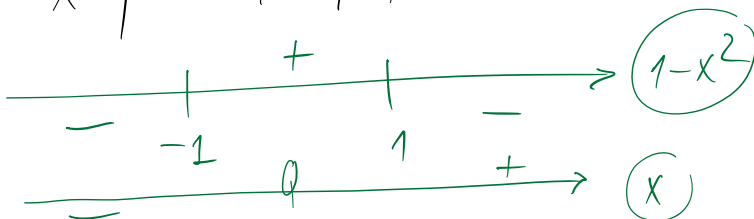
Signe de x

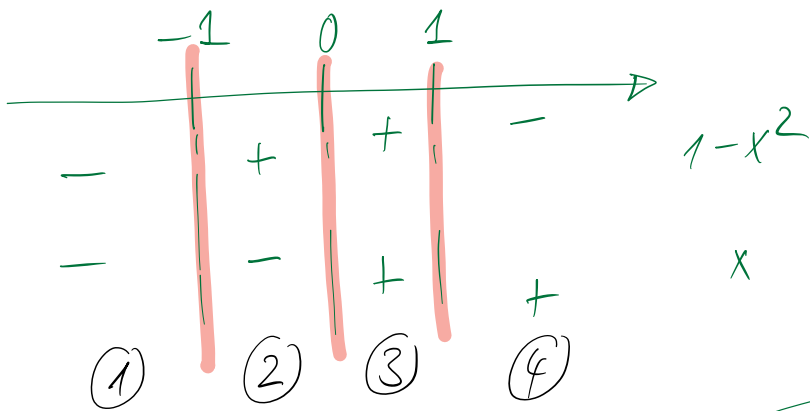


∩

$$(1-x)(1+x) = 0 \Leftrightarrow x = \pm 1$$

$$|1-x^2| = 1 + |x|$$





$$\underline{1-x^2} = 1 + |x|$$

$$\textcircled{1} \quad 1-x^2 < 0 \text{ et } x < 0$$

$$-(1-x^2) = 1 - x$$

$$x^2 - 1 = 1 - x$$

$$x^2 + x - 2 = 0$$

$$\textcircled{2} \quad x \geq -1 \text{ et } x < 0$$

$$1-x^2 = 1-x$$

$$x^2 - x = 0$$

$$x = 1 \text{ et } x = 0$$

$$\textcircled{3} \quad x \geq 0 \text{ et } x \leq 1$$

$$1-x^2 = 1+x$$

$$x^2 + x = 0$$

$$x = -1 \text{ et } x = 0$$

$$\begin{cases} |1-x^2| = -(1-x^2) \\ \Rightarrow \text{ver } (1-x^2) < 0 \end{cases}$$

$$\text{ver } x < -1 \Rightarrow |x| = -x \text{ ver } x < 0$$

$$x = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} \begin{cases} 1 \\ -2 \end{cases}$$

OK

$$(4) \quad x > 1$$

$$-(1-x^2) = 1+x$$

$$x^2 - 1 = 1+x$$

$$x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{9}}{2} = \begin{cases} (2) \text{ OK} \\ -1 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & \text{si } x-2 \geq 0 \\ -(x-2) & \text{si } x-2 < 0 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & \text{si } x \geq 2 \\ \underbrace{-(x-2)}_{-x+2} & \text{si } x < 2 \end{cases}$$

$$\begin{aligned} \sqrt{x+2} + \sqrt{x-2} &= \sqrt{2x+3} + \sqrt{2x-3} && 2.5.14 \\ (A+B)^2 &= A^2 + 2AB + B^2 && i) \\ \downarrow &&& \downarrow \end{aligned}$$
$$x+2 + 2\sqrt{x+2}\sqrt{x-2} + x-2$$
$$x+2 + 2\sqrt{x^2-4} + x-2 = 2x+3 + 2\sqrt{4x^2-9} + 2x-3$$

$$2\sqrt{x^2-4} + 2x = 4x + 2\sqrt{4x^2-9}$$

$$\sqrt{x^2-4} = x + \sqrt{4x^2-9}$$

$$\sqrt{x^2-4} - \sqrt{4x^2-9} = x$$

$$\begin{array}{ccc} \left. \begin{array}{l} \downarrow \\ \downarrow \end{array} \right\} ()^2 & & \left. \begin{array}{l} \downarrow \\ \downarrow \end{array} \right\} ()^2 \\ x^2-4 - 2\sqrt{(x^2-4)(4x^2-9)} & & + 4x^2-9 = x^2 \end{array}$$

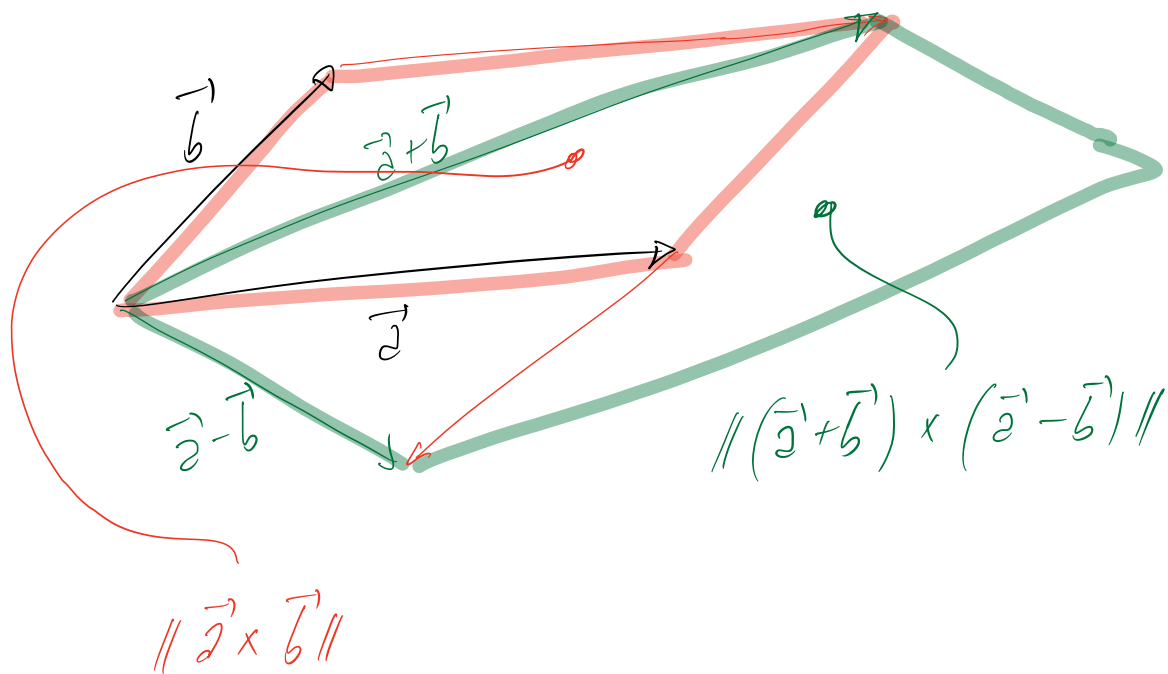
$$-2\sqrt{4x^4-9x^2-16x^2+36} = -4x^2+13$$

$$\left. \begin{array}{l} \downarrow \\ \downarrow \end{array} \right\} ()^2 \quad \left. \begin{array}{l} \downarrow \\ \downarrow \end{array} \right\} ()^2 \\ 4(4x^4-9x^2-16x^2+36) = 16x^4-104x^2+169$$

$$16x^4-36x^2-64x^2+144 = 16x^4-104x^2+169$$

$$4x^2 = 25 \quad x = \pm \frac{5}{2}$$

$$\text{On garde } x = \frac{5}{2}$$



$$\boxed{\vec{a} \cdot \vec{b}} = \frac{1}{4} \left(\|\vec{a} + \vec{b}\|^2 - \|\vec{a} - \vec{b}\|^2 \right)$$

$$= \frac{1}{4} \left((a_1 + b_1)^2 + (a_2 + b_2)^2 - \left((a_1 - b_1)^2 + (a_2 - b_2)^2 \right) \right)$$

$$= \frac{1}{4} \left(\cancel{a_1^2} + 2a_1b_1 + \cancel{b_1^2} + \cancel{a_2^2} + 2a_2b_2 + \cancel{b_2^2} - (a_1^2 - 2a_1b_1 + b_1^2 + a_2^2 - 2a_2b_2 + b_2^2) \right)$$

$$= \frac{1}{4} \left(4a_1b_1 + 4a_2b_2 \right) = \boxed{a_1b_1 + a_2b_2}$$

$$4 - |x+2| = 3(|x-1| - 1)$$

$$|x-1| = \begin{cases} x-1 & \text{si } x-1 \geq 0 / x \geq 1 \\ -(x-1) & \text{si } x-1 < 0 / x < 1 \end{cases}$$

$$\boxed{x \geq 1} \quad 4 - |x+2| = 3 \left(\overbrace{(x-1) - 1}^{x-2} \right)$$

$$\boxed{\text{si } x \geq 1, x+2 \geq 3, |x+2| = x+2}$$

$$\Rightarrow 4 - (x+2) = 3x - 6$$

$$\Rightarrow 4 - x - 2 = 3x - 6$$

$$8 = 4x$$

$$\boxed{x=2} \quad \checkmark$$

$$\boxed{x < 1} \quad 4 - |x+2| = 3 \left(-(x-1) - 1 \right)$$

$$\boxed{x \geq -2} \quad 4 - (x+2) = -3x$$

$$2x = -2 \quad \boxed{x = -1} \quad \checkmark$$

$$x < -2$$

$$4 + x + 2 = -3x$$

$$4x = -6$$

$$x = -\frac{3}{2} = -1.5 > -2$$