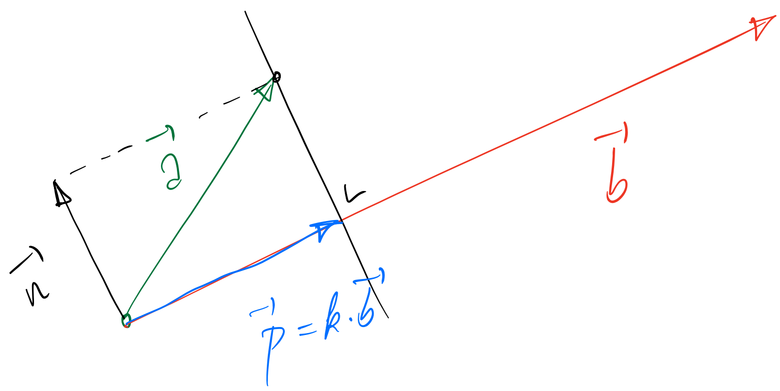
Projection



$$\vec{a} = \vec{n} + \vec{p} \quad \Big| \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = (\vec{n} + \vec{p}) \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = \underbrace{\vec{n} \cdot \vec{b}}_{=0 \text{ cor } \vec{n} \perp \vec{b}} + \vec{p}\vec{b} \quad \Rightarrow \quad \vec{a} \cdot \vec{b} = \vec{p} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = (k \cdot \vec{b}) \cdot \vec{b} = k \cdot (\vec{b} \cdot \vec{b})$$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \Rightarrow \vec{b} \cdot \vec{b} = b_1^2 + b_2^2 = \left(\sqrt{b_1^2 + b_2^2}\right)^2$$

$$= \|\vec{b}\|^2$$

$$\Rightarrow \quad \vec a \cdot \vec b = k \cdot \|\vec b\|^2 \qquad \boxed{k = \frac{\vec a \cdot \vec b}{\|\vec b\|^2}}$$

$$\Rightarrow \quad \vec p = k\,\vec b \quad \Rightarrow \quad \boxed{\vec p = \underbrace{\frac{\vec a \cdot \vec b}{\|\vec b\|^2}}_{k} \cdot \vec b}$$

vecteur

$$\boxed{\|\vec p\| = \frac{|\vec a \cdot \vec b|}{\|\vec b\|}}$$

nombre

$$|x| = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{sinon} \end{cases}$$

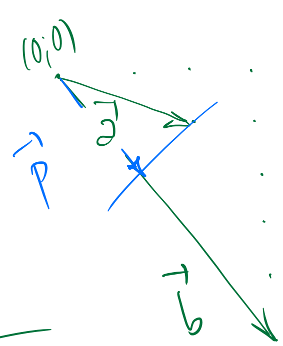
valeur absolue

Exemple: $\vec a = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\quad \vec b = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$$\vec p = \frac{\vec a \cdot \vec b}{\|\vec b\|^2}\,\vec b$$

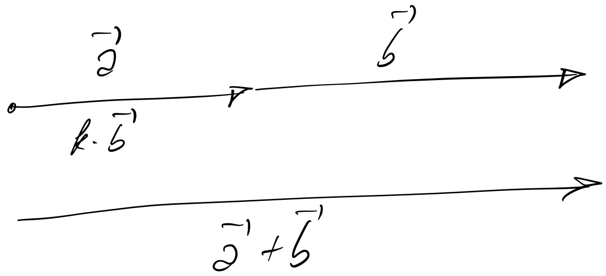
$$= \frac{2 \cdot 3 + (-1)\cdot(-5)}{9+25} \cdot \vec b$$

$$= \frac{11}{34} \cdot \vec b = \begin{pmatrix} 33/34 \\ -55/34 \end{pmatrix}$$



$$\|\vec p\| = \frac{11}{\sqrt{34}} \simeq 1{,}89$$

$$\|\vec{a}\| + \|\vec{b}\| \overset{?}{=} \|\vec{a}+\vec{b}\|$$



$$\|\vec{a}+\vec{b}\| = \|\vec{a}\| + \|\vec{b}\|$$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\boxed{\vec{a} = k\cdot\vec{b} \quad \text{avec} \quad k>0}$$

$$\|\vec{a}\| + \|\vec{b}\| \overset{\vec{a}=k\cdot\vec{b}}{=} \|k\vec{b}\| + \|\vec{b}\|$$

$$= \sqrt{k^2\cdot b_1^2 + k^2\cdot b_2^2} + \|\vec{b}\|$$

$$= \sqrt{k^2\cdot(b_1^2+b_2^2)} + \|\vec{b}\|$$

$$\overset{k>0}{=} k\cdot\sqrt{b_1^2+b_2^2} + \sqrt{b_1^2+b_2^2}$$

$$= (k+1)\,\|\vec{b}\|$$

$$\|\vec{a}+\vec{b}\| \overset{\vec{a}=k\vec{b}}{=} \|k\vec{b}+\vec{b}\| = \|(k+1)\cdot\vec{b}\| = \sqrt{(k+1)^2 b_1^2 + (k+1)^2\cdot b_2^2}$$

$$= \sqrt{(k+1)^2 (b_1^2 + b_2^2)} \underset{k>0}{=} (k+1)\sqrt{b_1^2 + b_2^2}$$

$$= (k+1)\,\|\vec{b}\|$$

On en déduit :

Si $\vec{a} = k \cdot \vec{b}$ avec $k > 0$, alors $\|\vec{a} + \vec{b}\| = \|\vec{a}\| + \|\vec{b}\|$