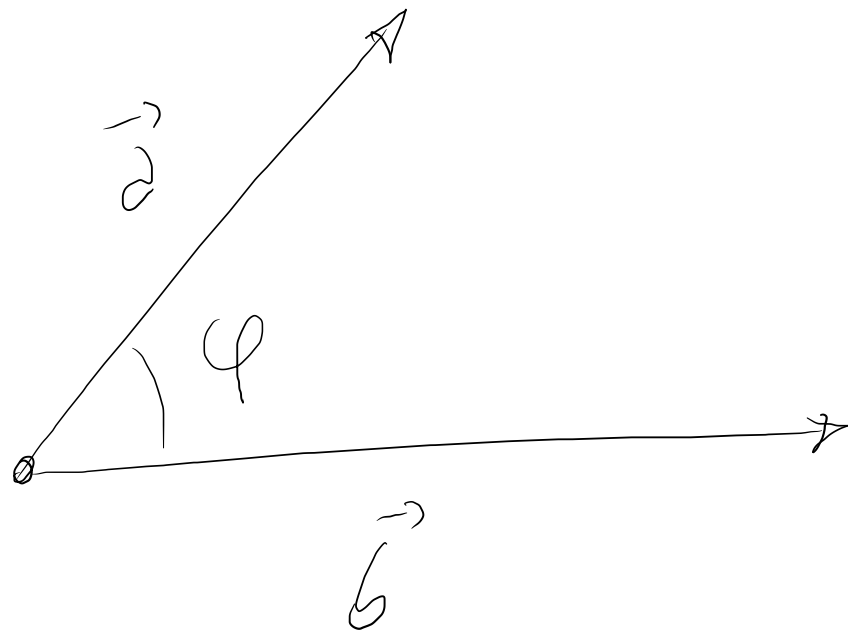
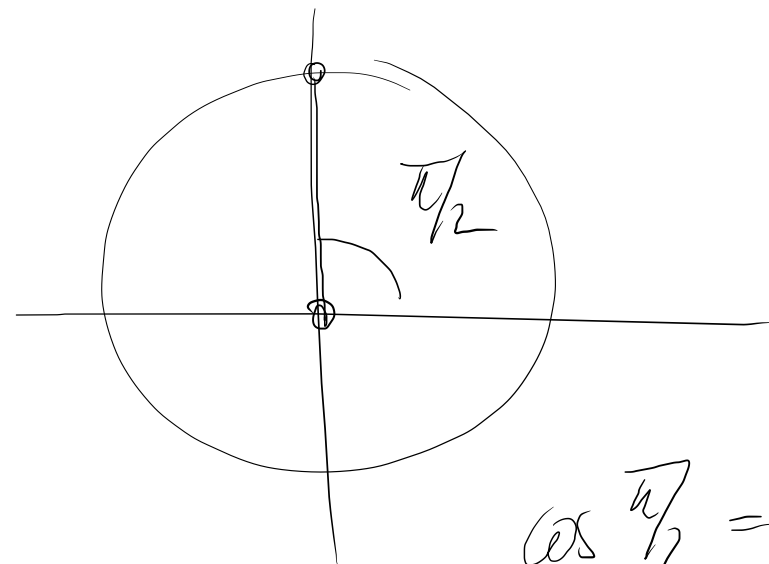
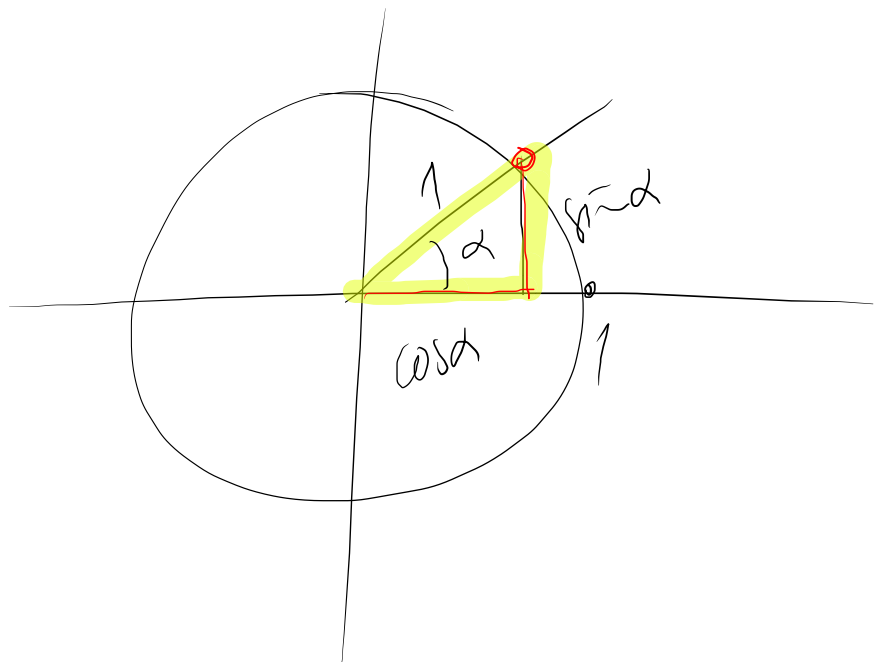


# Produit scalaire et angles



$$\cos \varphi = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

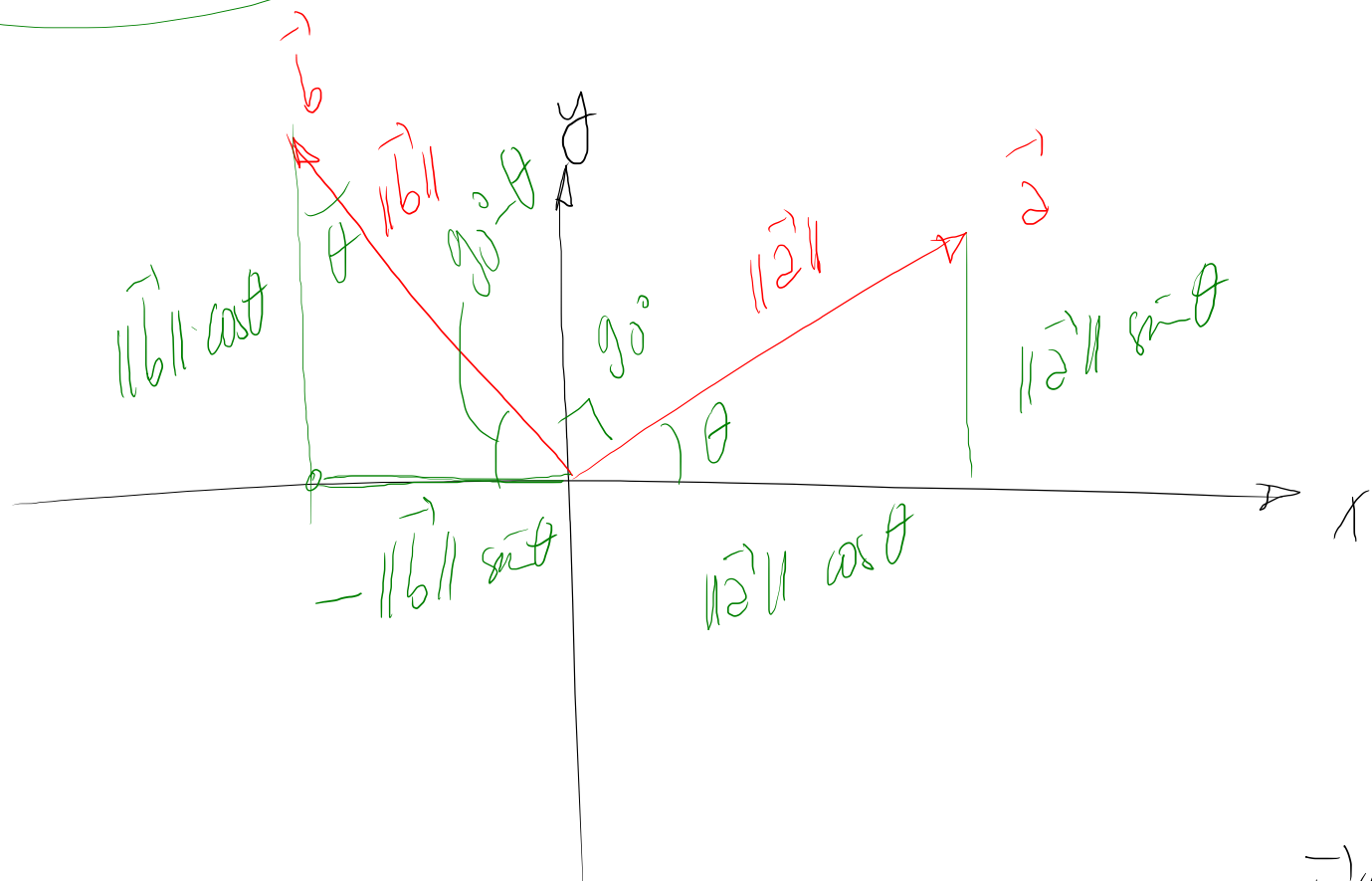
$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \stackrel{\text{def}}{=} a_1 \cdot b_1 + a_2 \cdot b_2$$



$$\cos \frac{\pi}{2} = 0$$

$$\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \perp \vec{b}$$



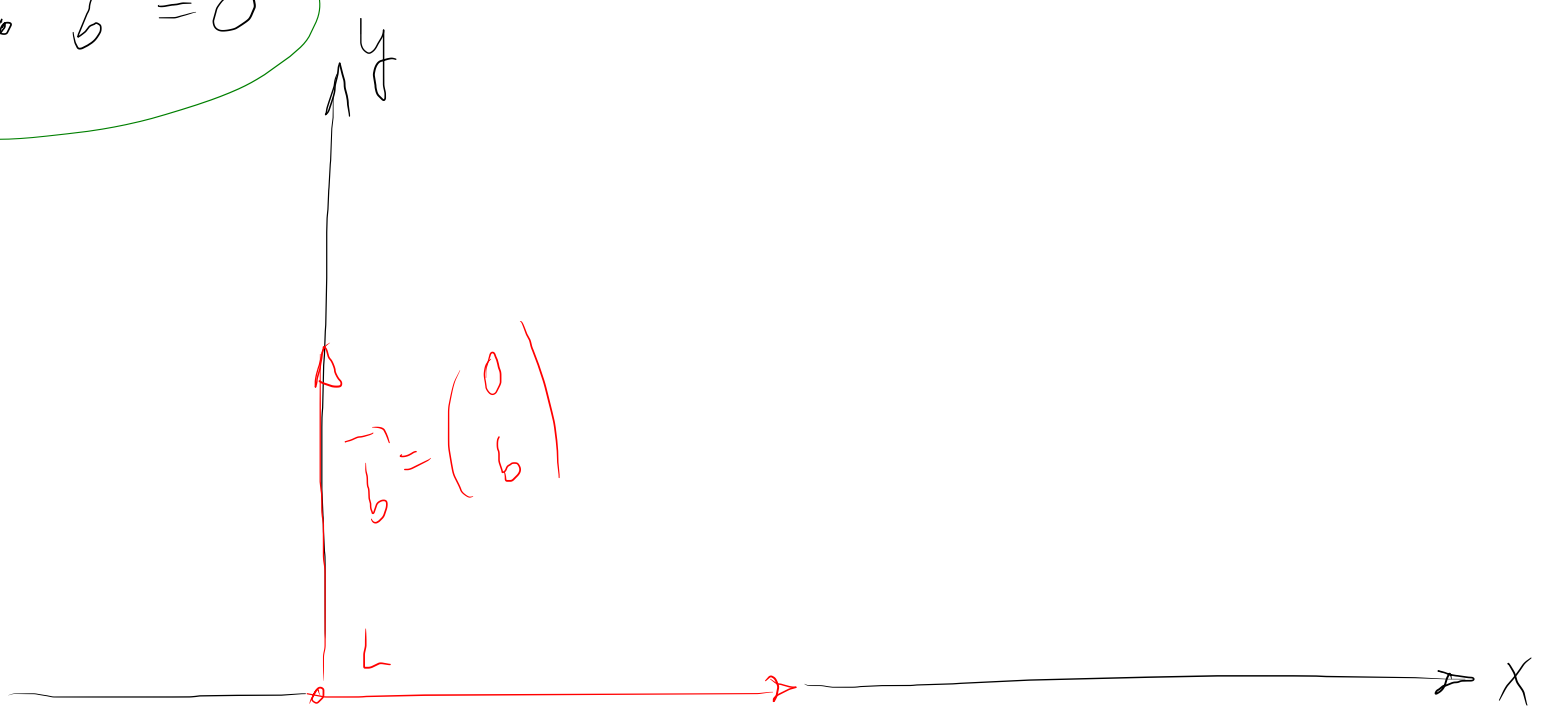
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\|\vec{b}\|}{\|\vec{a}\|}$$

$$\vec{a} = \begin{pmatrix} \|\vec{a}\| \cdot \cos \theta \\ \|\vec{a}\| \cdot \sin \theta \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} -\|\vec{b}\| \sin \theta \\ \|\vec{b}\| \cos \theta \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = -\|\vec{a}\| \|\vec{b}\| \cos \theta \sin \theta + \|\vec{a}\| \|\vec{b}\| \sin \theta \cos \theta = 0$$

$$\vec{a} \cdot \vec{b} = 0$$



$$\vec{b} = \begin{pmatrix} 0 \\ b \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

Sei  $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a \\ 0 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \boxed{a b_1 = 0}$$

$a = 0$  or  $b_1 = 0 \Rightarrow \vec{b} = \begin{pmatrix} 0 \\ b \end{pmatrix}$   
 $\uparrow$   
absurde

$$\Rightarrow \vec{a} \perp \vec{b}$$