Définition : Un vecteur est dit <u>unitaire</u> si sa norme vaut 1.

Exemple : $\vec{u} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$ est unitaire.

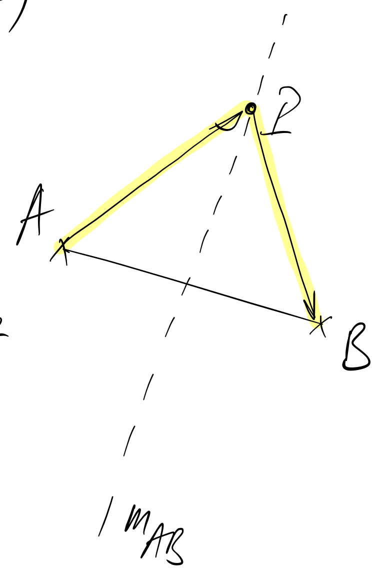
En effet, $\|\vec{u}\| = \left[\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]^{\frac{1}{2}} = \left(\frac{3}{4} + \frac{1}{4}\right)^{\frac{1}{2}}$

$= \left(\frac{4}{4}\right)^{\frac{1}{2}} = \sqrt{1} = 1$

$$\|\vec{u}\| = \left\|\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\right\| = \sqrt{u_1^2 + u_2^2}$$

$$\|\vec{u}\| = \left\|\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}\right\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$A(5;3)$ $\quad B(-2;k)$ $\quad P(2;-1)$



$$\boxed{\|\overrightarrow{AP}\|^2 = \|\overrightarrow{PB}\|^2}$$

$$(2-5)^2 + (-1-3)^2 = (-4)^2 + (k+1)^2$$

$$(k+1)^2 = 9$$

$$k^2 + 2k + 1 = 9$$

$$k^2 + 2k - 8 = 0$$

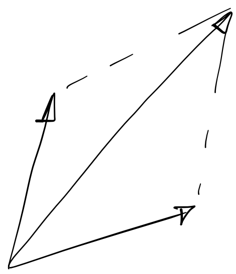
$$(k+4)(k-2) = 0$$

$$k = -4 \;/\; k = 2$$

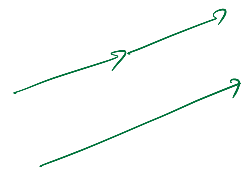
$$\|u\| = \|v\| \iff \|u\|^2 = \|v\|^2$$

$$\sqrt{182} = \sqrt{2 \cdot 91} = \sqrt{2 \cdot 7 \cdot 13}$$

1.4.8



$\vec{a}, \vec{b}$ de dim 2



$\boxed{\vec{a}, \vec{b} \text{ doivent être colinéaires.}}$

et de même sens.

$\vec{a} = k\vec{b} \Rightarrow \quad \|\vec{a} + \vec{b}\| = \|k\vec{b} + \vec{b}\|$

$$= \|(k+1)\vec{b}\| = \left\| \begin{pmatrix} (k+1)b_1 \\ (k+1)b_2 \end{pmatrix} \right\|$$

$$= \sqrt{(k+1)^2 b_1^2 + (k+1)^2 b_2^2}$$

$\sqrt{XY} = \sqrt{X}\sqrt{Y}$

$X, Y \geq 0$

$$= \sqrt{(k+1)^2 \cdot (b_1^2 + b_2^2)}$$

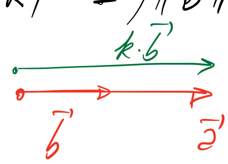
$$= \sqrt{(k+1)^2} \cdot \sqrt{b_1^2 + b^2}$$

$$= |k+1| \, \|\vec{b}\|$$

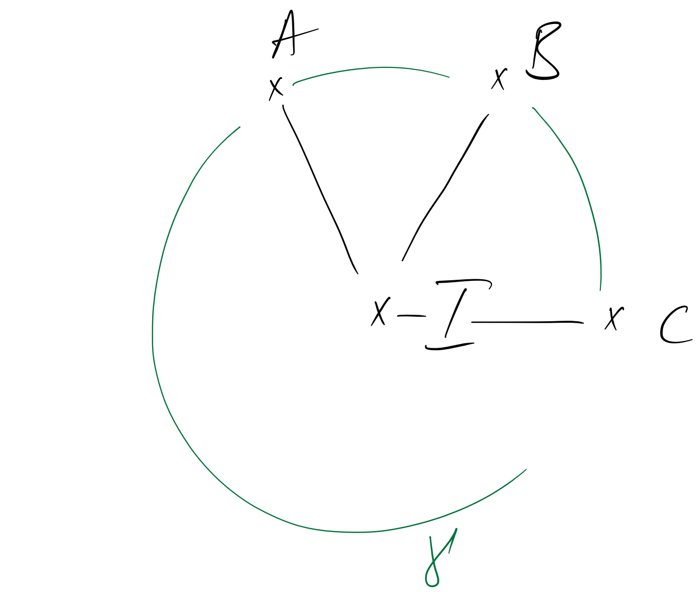
$$\|\vec{a}\| + \|\vec{b}\| = \|k\vec{b}\| + \|\vec{b}\| = \sqrt{k^2 \cdot b_1^2 + k^2 b_2^2} + \sqrt{b_1^2 + b_2^2}$$

$$= |k| \cdot \|\vec{b}\| + \|\vec{b}\| = (|k| + 1)\|\vec{b}\|$$

Si $\vec{a}$ et $\vec{b}$ ont même sens, $k > 0$



$\Rightarrow \|\vec{a} + \vec{b}\| = \|\vec{a}\| + \|\vec{b}\|$



$$\|\vec{AI}\| = \|\vec{BI}\| = \|\vec{CI}\|$$

$$\iff A, B, C \in \gamma(I; r)$$

circle

$$A + k\vec{AB} = \vec{CD} \cdot l + C$$

$$\begin{pmatrix} 6 \\ 4 \\ -4 \end{pmatrix} + k\begin{pmatrix} -2 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \cdot l + \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix}$$

$$\begin{cases} 6 - 2k = 4l + 7 \\ 4 - 4k = -4l \\ -4 + 2k = 2l - 2 \end{cases} \iff \begin{cases} -2k = 4l + 1 \\ -4k = -4l - 4 \\ -4 + 2k = 2l - 2 \end{cases}$$

$$\iff \begin{cases} k = -2l - \frac{1}{2} \\ k = l + 1 \\ -4 + 2k = 2l - 2 \end{cases}$$

Deux vecteurs $\vec{a}, \vec{b}$ sont perpendiculaires

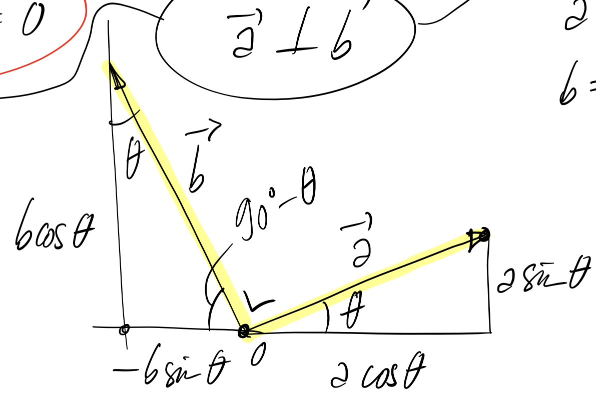
$$\Leftrightarrow \quad \vec{a} \cdot \vec{b} = 0 \qquad \vec{a} \perp \vec{b}$$

$a = \|\vec{a}\|$

$b = \|\vec{b}\|$

$$\vec{b} = \begin{pmatrix} -b\sin\theta \\ b\cos\theta \end{pmatrix}$$

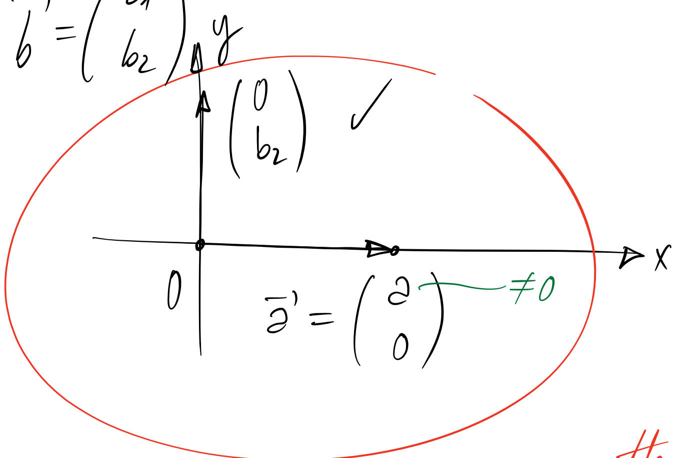
$$\vec{a} = \begin{pmatrix} a\cos\theta \\ a\sin\theta \end{pmatrix}$$



$$\vec{a} \cdot \vec{b} = -ab\cos\theta\sin\theta + ab\cos\theta\sin\theta = 0$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$



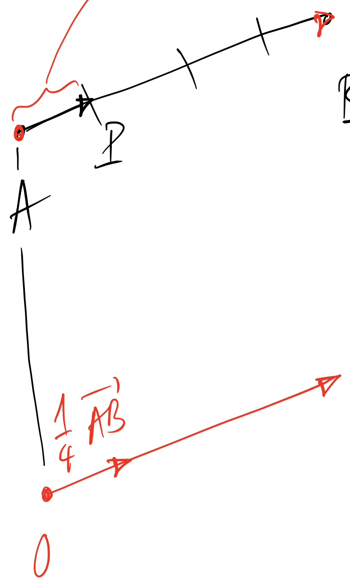
$$\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \begin{pmatrix} a \\ 0 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = ab_1 + 0 \cdot b_2$$

$$= ab_1 = 0$$

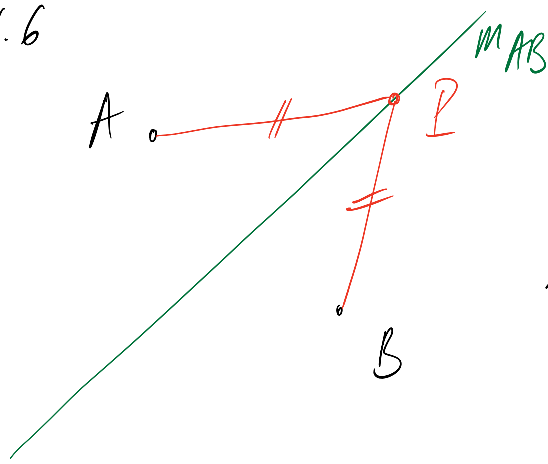
$$\Rightarrow b_1 = 0$$

On peut tjrs supposer cette configuration



$$\vec{OP} = \vec{OA} + \underbrace{\frac{1}{4}\vec{AB}}$$

1.4.6



$$\|\vec{AP}\| = \|\vec{BP}\|$$

$$\Leftrightarrow \sqrt{(x-1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y-8)^2}$$

$$\Leftrightarrow (x-1)^2+(y-2)^2 = (x-3)^2+(y-8)^2$$