

Prop. Soit $a, b, c \in \mathbb{R}$ avec $a \neq 0$.

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

preuve: $ax^2 + bx + c = 0$

$$\Leftrightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Leftrightarrow x^2 + 2 \cdot x \cdot \frac{b}{2a} + \frac{c}{a} = 0$$

$$\Leftrightarrow x^2 + 2 \cdot x \cdot \frac{b}{2a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a^2} - \frac{c}{a}\right) = 0$$

$$\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right) = 0$$

$$\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2 = 0$$

$$\Leftrightarrow \left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right) \left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right) = 0$$

$$\Leftrightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CQFD