$x^n y^0 \quad x^{n-1} y^1 \ \ldots \ x^{n-k} y^k \ \ldots \ x^0 y^n$

Prop. $n \in \mathbb{N}$, $x, y \in \mathbb{R}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

somme

```
S = 0
k = 0
while k < n:
    S += binomial(n, k) x**(n-k) *y** k
    k += 1
```

$$(x+y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n$$

Exemple:

$$(x+y)^5 = \underset{1}{\binom{5}{0}} x^5 + \underset{5}{\binom{5}{1}} x^4 y + \underset{10}{\binom{5}{2}} x^3 y^2 + \underset{10}{\binom{5}{3}} x^2 y^3 + \underset{5}{\binom{5}{4}} x y^4 + \underset{1}{\binom{5}{5}} y^5$$

5ème ligne du △ de Pascal

Preuve par réc. sur n

$$\frac{0!}{0!\,0!} = 1$$

$\boxed{n=0}$

$$(x+y)^0 \overset{?}{=} \binom{0}{0} \cdot x^0$$

$$1 = 1 \checkmark$$

$\boxed{n=1}$

$$(x+y)^1 = x+y$$

$$\binom{1}{0} x^1 y^0 + \binom{1}{1} x^0 y^1 = 1 \cdot x + 1 \cdot y = x+y$$

$$x+y = x+y \checkmark$$

$$\boxed{n \checkmark \Rightarrow n+1 \checkmark}$$

Hyp. de réc.

$$(x+y)^n = \boxed{\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k} \checkmark$$

$$(x+y)^{n+1} = (x+y)^n \cdot (x+y)$$

$$= \left[\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k\right] \cdot (x+y)$$

$$= \left(\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k\right) \cdot x + \left(\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k\right) \cdot y$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k+1} y^k + \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^{k+1}$$

$$\left[\binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{n} y^n\right] \cdot x = \binom{n}{0} x^{n+1} + \binom{n}{1} x^n y + \ldots + \binom{n}{n} x y^n$$

$$= \binom{n}{0} x^{n+1} + \binom{n}{1} x^n y + \ldots + \binom{n}{n-1} x^2 y^{n-1} + \binom{n}{n} x y^n$$

$$\binom{n}{0} x^n y^1 + \ldots + \binom{n}{n-2} x^2 y^{n-1} + \binom{n}{n-1} x y^n + \binom{n}{n} y^{n+1}$$

$$= \binom{n+1}{0} x^{n+1} + \binom{n+1}{1} x^n y + \ldots + \binom{n+1}{n-1} x^2 y^{n-1} + \binom{n+1}{n} x y^n + \binom{n+1}{n+1} y^{n+1}$$

$$= (x+y)^{n+1} = \sum_{k=0}^{n+1} \binom{n+1}{k} x^{n+1-k} y^k \qquad \text{CQFD}$$

<u>Remarque:</u> $\binom{n}{0} = \binom{m}{0} = 1 \quad \forall\ n, m \geq 0$

$$\binom{n+1}{n+1} = \binom{n}{n} \quad \forall\ n$$