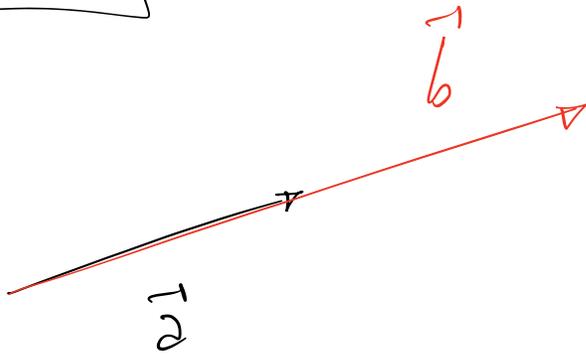


Colinéarité



$$\vec{a}, \vec{b} \text{ colinéaires} \Leftrightarrow \vec{a} = k \cdot \vec{b}$$

En dimension 2 :

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ et $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ sont colinéaires

$$\Leftrightarrow \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 = 0$$

si et seulement si

déterminant d'une matrice 2x2

Exemple : $\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = 4 - 6 = -2$

preuve: $\Rightarrow \vec{a} = k \vec{b} \Rightarrow \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = k \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} kb_1 \\ kb_2 \end{pmatrix}$

$$\Rightarrow \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} kb_1 & b_1 \\ kb_2 & b_2 \end{vmatrix} = kb_1 b_2 - kb_2 b_1$$

$$= kb_1 b_2 - kb_1 b_2$$

$\Leftarrow \vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = 0$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0 \Rightarrow a_1 b_2 - a_2 b_1 = 0$$

$$\Rightarrow a_1 b_2 = a_2 b_1$$

① $b_2 = 0 \Rightarrow a_2 b_1 = 0 \Rightarrow b_1 = 0$ ou $a_2 = 0$ $b_1 \neq 0$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

colinéaires

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ 0 \end{pmatrix}, \begin{pmatrix} b_1 \\ 0 \end{pmatrix}$$

colinéaires

$$\vec{a} = k \cdot \vec{b}$$

$$k = \frac{a_1}{b_1}$$

$$\textcircled{2} \boxed{b_2 \neq 0}$$

$$a_1 = a_2 \cdot \frac{b_1}{b_2}$$

$$\vec{a} = \begin{pmatrix} a_2 \cdot \frac{b_1}{b_2} \\ a_2 \end{pmatrix} = \begin{pmatrix} a_2 \cdot \frac{b_1}{b_2} \\ a_2 \cdot \frac{b_2}{b_2} \end{pmatrix}$$

$$\begin{aligned} a \cdot \frac{b}{c} &= \frac{ab}{c} \\ &= \frac{a}{c} \cdot b \end{aligned}$$

↓

$$= \begin{pmatrix} \frac{a_2}{b_2} \cdot b_1 \\ \frac{a_2}{b_2} \cdot b_2 \end{pmatrix} = \frac{a_2}{b_2} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Rightarrow \vec{a} = \frac{a_2}{b_2} \cdot \vec{b} = k \cdot \vec{b}$$

CQFD