$$P(x) = \sum_{i=0}^{n} a_i \cdot x^i$$

$$= a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

Exemples :

$$P(x) = \boxed{3x^2 - 2x + 3} = \underbrace{3}_{a_0} \underbrace{-2}_{a_1}x + \underbrace{3}_{a_2}x^2 \qquad n = 2$$

$$Q(x) = 4x^3 - 3x^2 + 2x - 1$$

$\boxed{\text{Calculer } P(a) \text{ avec } a \in \mathbb{R}.}$ $\qquad x \mapsto a \;/\; \underbrace{3a^2 - 2a + 3}_{P(a)}$

$$P(0) = 3 \cdot 0^2 - 2 \cdot 0 + 3 = 3$$

$$P(1) = 3 \cdot 1^2 - 2 \cdot 1 + 3 = 4$$

$$P(25) = 1828$$

$$73 \cdot 100 = 7300$$

$$= 1825$$

$$\begin{array}{c|ccc} & 3 & -2 & 3 \\ 25 & & 75 & 1825 \\ \hline & 3 & 73 & 1828 \end{array}$$

$$\overset{?}{=} 3 \cdot 25^2 - 2 \cdot 25 + 3$$

$$= 3 \cdot 625 - 50 + 3 = 1828$$

$$Q(x) = \boxed{4x^3 - 3x^2 + 2x - 1}$$

$$Q(5) = 4\cdot 125 - 3\cdot 25 + 10 - 1 = 500 - 75 + 10 - 1$$

$$= 425 + 10 - 1 = 434$$

	4	−3	2	−1
5		20	85	435
	4	+17	87	434

$$Q(8) = \boxed{4\cdot 8^3 - 3\cdot 8^2 + 2\cdot 8 - 1}$$

$$\underset{2048}{4\cdot 8^3} - 192 + 16 - 1 = 1871$$

468

936

1872

	4	−3	2	−1
8		32	232	1872
	4	29	234	1871

$\cdot 8$ $\cdot 8$ $\cdot 8$

Schéma de Horner

$$\begin{array}{r|l} P(x) & Q(x) \\ \cline{2-2} & B(x) \\ \cline{1-1} R(x) & \end{array}$$

Div. eucl.

$$P(x) = Q(x) \cdot B(x) + R(x)$$

Cas particulier $\quad a \in \mathbb{R}$

$$\begin{array}{r|l} P(x) & x-a \\ \cline{2-2} & B(x) \\ \cline{1-1} R(x) & \end{array}$$

$$P(x) = (x-a)\, B(x) + R(x)$$

$$P(a) = \underbrace{(a-a)}_{0} \cdot B(x) + R(a)$$

$$\boxed{P(a) = R(a) = R}$$

$\deg R < \deg(x-a) \Rightarrow R \in \mathbb{R}$

$$
\begin{array}{l|l}
\overbrace{4x^3 - 3x^2 + 2x - 1}^{P(x)} & x - 8 \\
\hline
4x^3 - 32x^2 & 4x^2 + 29x + 234 \\
\hline
\quad 29x^2 + 2x & \\
\quad 29x^2 - 232x & \\
\hline
\qquad 234x - 1 & \\
\qquad 234x - 1872 & \\
\hline
\qquad\quad 1871 & \\
\end{array}
$$

$P(8)$