

Deux trucs

$$x^4 + 1 = x^4 + 2x^2 + 1 - 2x^2$$

$$= (x^2 + 1)^2 - (\sqrt{2} \cdot x)^2$$

$$= (x^2 + 1 + \sqrt{2} \cdot x)(x^2 + 1 - \sqrt{2} \cdot x)$$

$$= (x^2 + \sqrt{2} \cdot x + 1)(x^2 - \sqrt{2} \cdot x + 1)$$

$$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$$

$$= (x^2 + 1)^2 - x^2 = (x^2 + x + 1)(x^2 - x + 1)$$

$$\boxed{x^2 + 1}$$

Irreducible

$$\sqrt{2} \cdot x \cdot \sqrt{2} \cdot x = \underbrace{\sqrt{2} \cdot \sqrt{2}}_2 \cdot \underbrace{x \cdot x}_{x^2} = 2x^2$$

$$x^4 + 1 \stackrel{!}{=} \underbrace{x^4 + 2x^2 + 1}_{(x^2+1)^2} - 2x^2 = (x^2+1)^2 - (\sqrt{2} \cdot x)^2$$

$A^2 - B^2$

$$= \underbrace{(x^2+1+\sqrt{2} \cdot x)}_{\text{Irreducible}} \underbrace{(x^2+1-\sqrt{2} \cdot x)}_{\text{Irreducible}}$$

$$x^2 + \sqrt{2}x + 1$$

Irreducible

$$\Delta = 2 - 4 \cdot 1 \cdot 1 = -2$$

$$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$$

$$= (x^2+1)^2 - x^2 = (x^2+x+1)(x^2-x+1)$$

$$8y^3(y-1) + 1 \cdot (y-1) = (8y^3 + 1)(y-1)$$

$$8y^3 = 2 \cdot 2 \cdot 2 \cdot y \cdot y \cdot y$$

$$= 2^3 \cdot y^3 = (2y)^3$$

$$(y-1) \cdot \textcircled{?} = (y-1)$$

↑
1

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2) = ((2y)^3 + 1^3)(y-1)$$

$$A = 2y \quad B = 1$$

$$= (2y + 1)(4y^2 - 2y + 1)(y-1)$$

$$\Delta = 4 - 4 \cdot 4 \cdot 1 < 0$$

$$12a^3 + \frac{9ab^2}{4} + \frac{3b^3}{16} + 9a^2b =$$

$$12 \left(a^3 + \frac{3}{16} ab^2 + \frac{3}{4} a^2b + \frac{b^3}{64} \right) =$$

$$12 \left(a^3 + 3 \cdot a \cdot \left(\frac{b}{4} \right)^2 + 3 \cdot a^2 \cdot \left(\frac{b}{4} \right) + \left(\frac{b}{4} \right)^3 \right) =$$

$$12 \left(a + \frac{b}{4} \right)^3$$