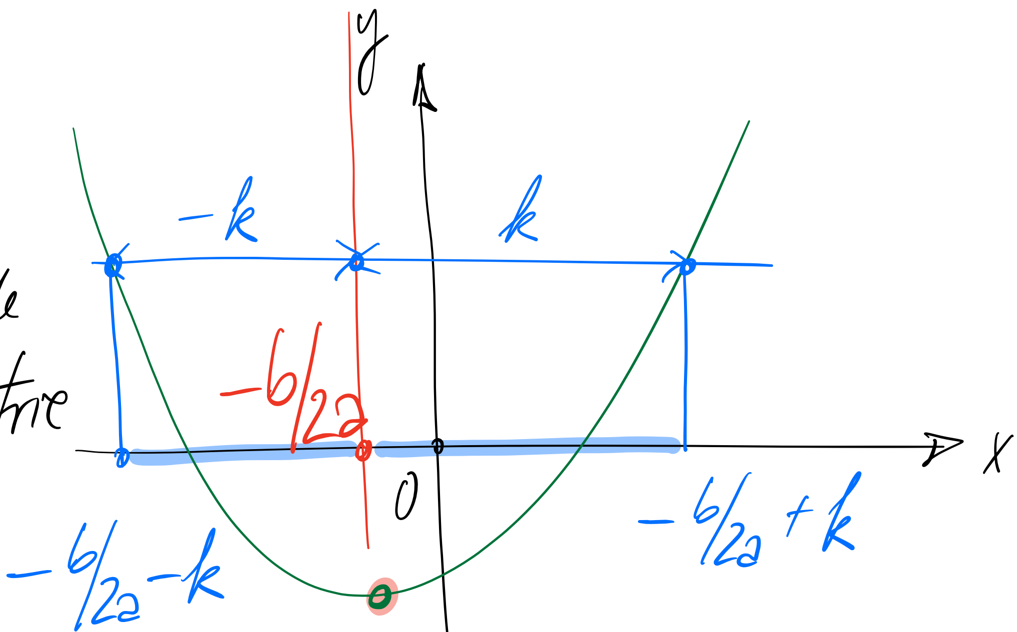
$$f(x) = ax^2 + bx + c \qquad a \neq 0$$

Prop. f admet un axe de symétrie



Preuve:

Il suffit de démontrer que $f(-b/2a - k) = f(-b/2a + k)$

$$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$A^2 + 2AB + B^2 = (A+B)^2$

$$x^2 + 2 \cdot x \cdot \frac{b}{2a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}$$

A B

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a^2} - \frac{c}{a}\right)\right)$$

$$= \boxed{a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right)}$$

Soit $k \geq 0$, $k \in \mathbb{R}$

$$f(-b/2a + k) = a\left(\left(-\cancel{\frac{b}{2a}} + k + \cancel{\frac{b}{2a}}\right)^2 - \frac{\Delta}{4a^2}\right)$$

$$= a\left(k^2 - \frac{\Delta}{4a^2}\right)$$

$$f\left(-\frac{b}{2a} - k\right) = a\left(\left(-\cancel{\frac{b}{2a}} - k + \cancel{\frac{b}{2a}}\right)^2 - \frac{\Delta}{4a^2}\right)$$

$$= a\left((-k)^2 - \frac{\Delta}{4a^2}\right) = a\left(k^2 - \frac{\Delta}{4a^2}\right)$$

$$\Rightarrow f\left(-\frac{b}{2a} + k\right) = f\left(-\frac{b}{2a} - k\right)$$ CQFD

<u>Application</u> : $S(1; -2)$ est le sommet d'une parabole

$$f(x) = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right)$$

$$1 = -\frac{b}{2a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a} \qquad -2 = -\frac{\Delta}{4a^2}$$

$$-\frac{b}{2a} = 1$$

$$\boxed{f(x) = a(x-1)^2 - 2}$$