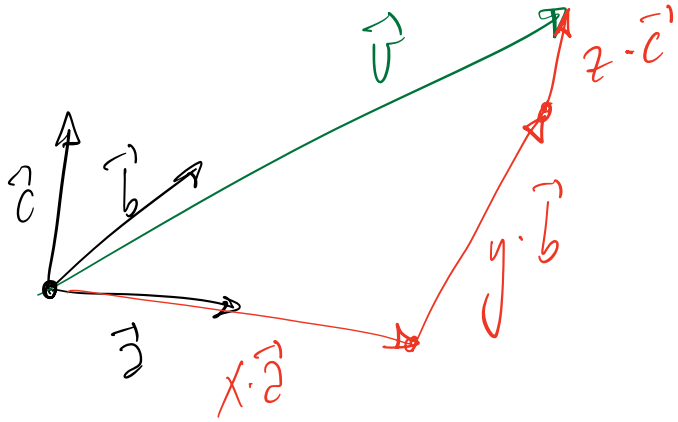
$$\vec{v} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \qquad \vec{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad \vec{c} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + z\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$$

$$\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} x \\ 2x \\ -x \end{pmatrix} + \begin{pmatrix} y \\ y \\ -y \end{pmatrix} + \begin{pmatrix} 2z \\ 2z \\ z \end{pmatrix}$$

Système 3x3

$$\begin{cases} 1 \cdot x + y + 2z = 3 \\ 2x + y + 2z = 1 \\ -x - y + z = -4 \end{cases}$$

Combinaisons linéaires

$L_2 \leftarrow L_2 - 2L_1$

$L_3 \leftarrow L_3 + L_1$

$$\begin{array}{r} -2x - 2y - 4z = -6 \\ + \quad 2x + y + 2z = 1 \\ \hline 0x \quad -y - 2z = -5 \end{array}$$

$$\begin{cases} x + y + 2z = 3 \\ -y - 2z = -5 \\ 3z = -1 \end{cases} \Rightarrow z = -\frac{1}{3}$$

$$\begin{array}{r} x + y + 2z = 3 \\ -x - y + z = -4 \\ \hline 3z = -1 \end{array}$$

$$\Rightarrow -y - 2\cdot(-1/3) = -5$$

$$y = 5 + \frac{2}{3} = \boxed{\frac{17}{3}}$$

$$\Rightarrow x + \frac{17}{3} - \frac{2}{3} = 3$$

$$x + 5 = 3$$

$$x = \boxed{-2}$$

$$\boxed{\vec{v} = -2\vec{a} + \frac{17}{3}\vec{b} - \frac{1}{3}\vec{c}}$$

$$\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\cdot\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)=0$$

$$X\cdot Y=0 \Rightarrow X=0 \text{ or } Y=0$$

$$x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}=0$$

$$x=-\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b+\sqrt{b^2-4ac}}{2a}}$$

$$x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}=0$$

$$\boxed{x=-\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b-\sqrt{b^2-4ac}}{2a}}$$

1.2.14

a) $x\begin{pmatrix}3\\5\\2\end{pmatrix} + y\begin{pmatrix}4\\-8\\6\end{pmatrix} + z\cdot\begin{pmatrix}-16\\10\\7\end{pmatrix} = \begin{pmatrix}0\\0\\52\end{pmatrix}$

$$\Leftrightarrow \begin{cases} 3x + 4y - 16z = 0 \\ 5x - 8y + 10z = 0 \\ 2x + 6y + 7z = 52 \end{cases} \qquad \begin{matrix} \\ L_2 \leftarrow 3L_2 \\ L_3 \leftarrow 3L_3 \end{matrix}$$

$$\begin{cases} 3x + 4y - 16z = 0 \\ 15x - 24y + 30z = 0 \\ 6x + 18y + 21z = 156 \end{cases} \qquad \begin{matrix} \\ L_2 \leftarrow L_2 - 5L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{matrix}$$

$$\begin{cases} 3x + 4y - 16z = 0 \\ -44y + 110z = 0 \\ 10y + 53z = 156 \end{cases} \qquad L_2 \leftarrow \frac{-1}{11} \cdot L_2$$

$$\begin{cases} 3x + 4y - 16z = 0 \\ 4y - 10z = 0 \\ 10y + 53z = 156 \end{cases} \qquad L_3 \leftarrow 4L_3$$

$$\begin{cases} 3x + 4y - 16z = 0 \\ 4y - 10z = 0 \\ 40y + 212z = 624 \end{cases} \qquad L_3 \leftarrow L_3 - 10L_2$$

$$\begin{cases} 3x + 4y - 16z = 0 \\ 4y - 10z = 0 \\ 312z = 624 \end{cases} \Rightarrow \quad z = 2$$

$$4y = 10 \cdot 2 = 20 \,/\, y = 5$$

$$\begin{cases} x = 4 \\ y = 5 \\ z = 2 \end{cases}$$

$$3x + 4 \cdot 5 - 16 \cdot 2 = 0$$

$$3x = 12 \,/\, x = 4$$

$$12a^3 + \frac{9}{4}a^2b + 9a^2b + \frac{3}{16}b^3$$

$$12a^3 + 9a^2b + \frac{9}{4}ab^2 + \frac{3}{16}b^3$$

$$A^3 + 3A^2B + 3AB^2 + B^3$$

$$\frac{\frac{9}{4}}{\frac{12}{1}} = \frac{9}{4} \cdot \frac{1}{12} = \frac{9}{4 \cdot 12}$$

$$12\left(a^3 + \frac{9}{12}a^2b + \frac{9}{48}ab^2 + \frac{3}{4}\cdot\frac{1}{16}b^3\right)$$

$$12\left(a^3 + \frac{9}{12}a^2b + \frac{9}{48}ab^2 + \frac{1}{64}b^3\right) =$$

$$12\left(a^3 + 3\cdot\frac{1}{4}a^2b + 3\cdot\frac{1}{16}ab^2 + \left(\frac{b}{4}\right)^3\right)$$

$$12\left(a^3 + 3\cdot a^2\frac{b}{4} + 3\cdot a\left(\frac{b}{4}\right)^2 + \left(\frac{b}{4}\right)^3\right)$$

$$12\left(a + \frac{b}{4}\right)^3$$

$$64x^6 - 91x^3 + 27 = 64\left(x^3 - 1\right)\left(x^3 - \frac{27}{64}\right)$$

$(x^3-1)(64x^3-27)$

$x^6 = (x^3)^2$ ✓

$\boxed{y = x^3} \Rightarrow \boxed{y^2 = x^6}$

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$= (x-1)(x^2+x+1)(4x-3)(16x^2+12x+9)$

$64y^2 - 91y + 27$

$$y = \frac{91 \pm \sqrt{91^2 - 4\cdot 64\cdot 27}}{128} = \frac{91 \pm 37}{128} \begin{cases} 1 \\ \frac{54}{128} = \frac{27}{64} \end{cases}$$

$A^3 - B^3$

$$64(x-1)\left(x^2+x+1\right)\left(x^3 - \left(\frac{3}{4}\right)^3\right) = \quad A = x \;\Big\}\; AB = \frac{3}{4}x \quad \left(\frac{3}{4}\right)^3$$

$B = \frac{3}{4}$

$$64(x-1)\left(x^2+x+1\right)\left(x - \frac{3}{4}\right)\left(x^2 + \frac{3}{4}x + \frac{9}{16}\right)$$

$(A-B)(A^2+AB+B^2)$

$$(A-B)\cdot(A^2+AB+B^2) = A^3+\cancel{A^2B}+\cancel{AB^2}$$
$$-\cancel{BA^2}-\cancel{AB^2}-B^3 = A^3-B^3$$