Division euclidienne

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

47	8
40	5
7	

~~$47 = 3 \cdot 8 + 23$~~

$47 = 8 \cdot 5 + 7$

$a, b \in \mathbb{Z}$ $\exists\, q, r$ $q \in \mathbb{Z}$ $r \in \mathbb{N}$

entiers relatifs

il existe

$$|x| = \begin{cases} -x & \text{si } x < 0 \\ x & \text{sinon} \end{cases}$$

tq. $a = b \cdot q + r$ $0 \leq r < |b|$

$$
\begin{array}{l|l}
3x^3-2x^2+4x-1 & x^2-2x+3 \\ \cline{2-2}
3x^3-6x^2+9x & 3x+4 \\ \cline{1-1}
\quad 4x^2-5x-1 & \\
\quad 4x^2-8x+12 & \\ \cline{1-1}
\quad\quad 3x-13 &
\end{array}
$$

$$x^2 \cdot ? = 3x^3$$

Egalité fondamentale : $3x^3-2x^2+4x-1 = \underbrace{(x^2-2x+3)(3x+4)}_{\text{quotient}} + \underbrace{3x-13}_{\text{reste}}$

$p, q \in \underbrace{\mathbb{Z}[x]}_{\text{polynômes}}$ $\exists\, s, r$ tq.

$$p = q \cdot s + r$$

et $\text{degré}(r) < \text{degré}(q)$

$\underbrace{x^4-1}_{p}, \underbrace{x+2}_{q}$ $\exists\, s, r$ tq.

$$x^4-1 = (x+2)\cdot s + r$$

$$
\begin{array}{l|l}
x^4+0\cdot x^3+0\cdot x^2+0\cdot x-1 & x+2 \\ \cline{2-2}
x^4+2x^3 & x^3-2x^2+4x-8 \\ \cline{1-1}
\quad -2x^3+0\cdot x^2 & \\
\quad -2x^3-4x^2 & \\ \cline{1-1}
\end{array}
$$

$$
\begin{array}{r}
4x^2 + 0 \cdot x \\
4x^2 + 8x \\
\hline
-8x - 1 \\
-8x - 16 \\
\hline
15
\end{array}
$$

reste

$$x^4 - 1 = (x+2)(x^3 - 2x^2 + 4x - 8) + 15$$