Suites de nombres

$$\sum_{k=0}^{n} \binom{k}{0} = 2^n$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

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1 2 3 4 5 6 ...

1 2 4 8 16 ...

1 3 5 7 9 ...

1 3 5 7 11 13 17 ...

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \ldots \longrightarrow 0$

$1 \quad 0{,}5 \quad 0{,}5^2 \quad 0{,}5^3 \quad 0{,}5^4$

$$(0{,}5)^n \xrightarrow[n\to\infty]{} 0$$

10^{-10}

$0{,}0000000001 \simeq 0$

$0{,}5^{33} \cong 1{,}16 \cdot 10^{-10}$

$0{,}5^{34} \cong 0{,}58 \cdot 10^{-10}$

$$\underbrace{0{,}5 \cdot 0{,}5 \cdot \ldots \_\_\_ \cdot 0{,}5}_{n\,=\,34} < $$

$$n = \log_{0{,}5}\left(10^{-10}\right) = \frac{LN\left(10^{-10}\right)}{LN(0{,}5)}$$

$$0{,}5^n = 10^{-10}$$

Soit $q \in \,]0;1[$

$$q^n \xrightarrow[n\to\infty]{?} 0$$

Preuve:

$q \in]0;1[\Rightarrow \exists t \in]0;1[$

tq. $q+t=1$ Il existe

Soit $n \in \mathbb{N}$

$$1 = 1^{n+1} = (q+t)^{n+1} = \sum_{k=0}^{n+1} \binom{n+1}{k} q^{n+1-k} t^k$$

$$\Rightarrow 1 = \binom{n+1}{0} q^{n+1} \cdot t^0 + \underbrace{\binom{n+1}{1}}_{n+1} q^n \cdot t^1 + \sum_{k=2}^{n+1} \binom{n+1}{k} q^{n+1-k} t^k$$

$$1 = \underbrace{q^{n+1}}_{>0} + \underbrace{\boxed{(n+1) \cdot q^n \cdot t}}_{>0} + \underbrace{\sum_{k=2}^{n+1} \binom{n+1}{k} q^{n+1-k} t^k}_{>0}$$

$$\Rightarrow (n+1) q^n \cdot t < 1$$

$\div (n+1) \cdot t$

$$q^n < \frac{1}{(n+1) \cdot t}$$

$$\Rightarrow \quad q^n < 0,\underbrace{00000\ldots000}_{p \text{ zéros}}1$$

$$\frac{1}{(n+1)\cdot t} < 0,0000\ldots0001$$

$$\Leftrightarrow \quad \frac{1}{0,0000\ldots0001\cdot t} < n+1$$