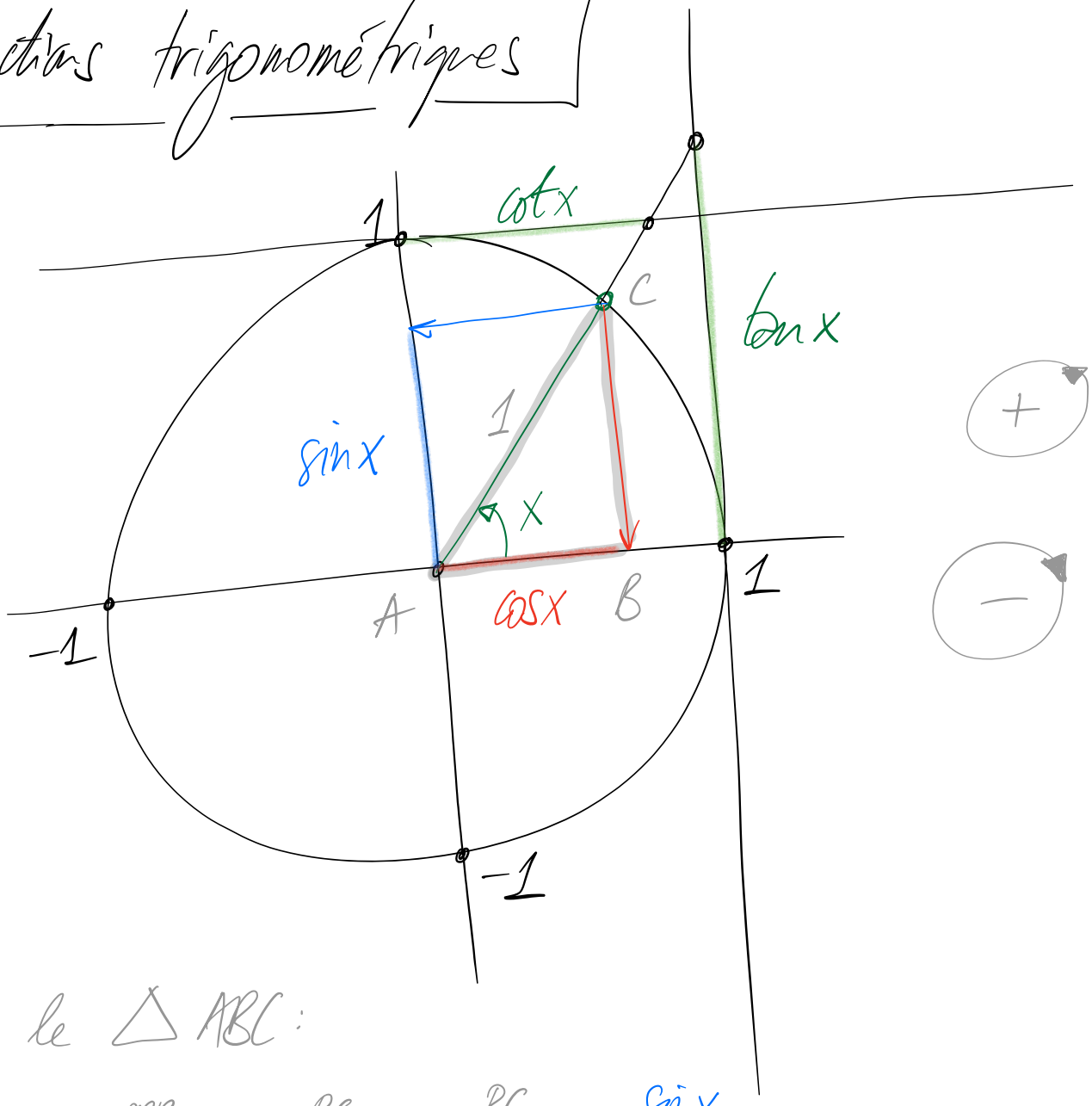
Fonctions trigonométriques



Dans le $\triangle ABC$:

$$\sin x = \frac{opp}{hyp} = \frac{BC}{AC} = \frac{BC}{1} = \frac{\sin x}{1}$$

$$\cos x = \frac{adj}{hyp} = \frac{AB}{AC} = \frac{AB}{1} = \frac{\cos x}{1}$$

Le thm de Pythagore donne : $AB^2 + BC^2 = AC^2 = 1^2$

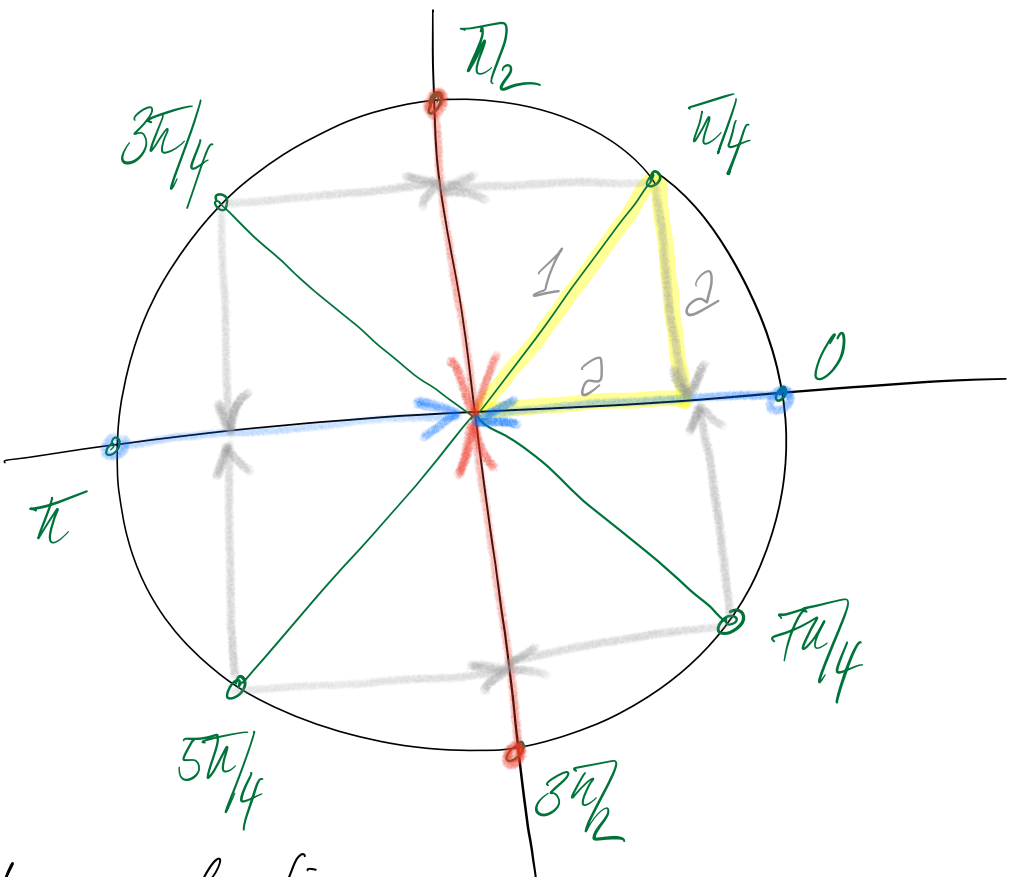
$$\Leftrightarrow \quad \cos^2 x + \sin^2 x = 1$$

Le thm de Thalès donne :

$$\frac{\tan x}{BC} = \frac{1}{AB} \iff \tan x = \frac{BC}{AB}$$

$$\iff \tan x = \frac{\sin x}{\cos x}$$

On a encore : $\cot x = \frac{1}{\tan x}$



On voit sur la figure :

$\cos \frac{\pi}{2} = \cos \frac{3\pi}{2} = 0$ $\qquad$ $\sin 0 = \sin \pi = 0$

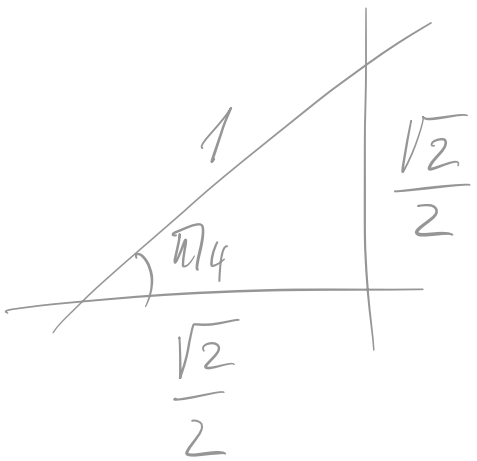
$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \sin\frac{3\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$$

En effet, $a^2+a^2=1 \iff 2a^2=1 \iff a^2=\frac{1}{2}$

$$\iff a = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\frac{3\pi}{4} = \cos\frac{5\pi}{4} = \sin\frac{5\pi}{4} = \sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$$

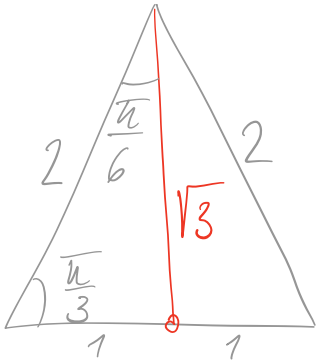
Dans le triangle isocèle et rectangle, dont la longueur de l'hypothénuse vaut 1, on a



$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan\frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$$

Dans le triangle équilatéral de côté 2, on a :

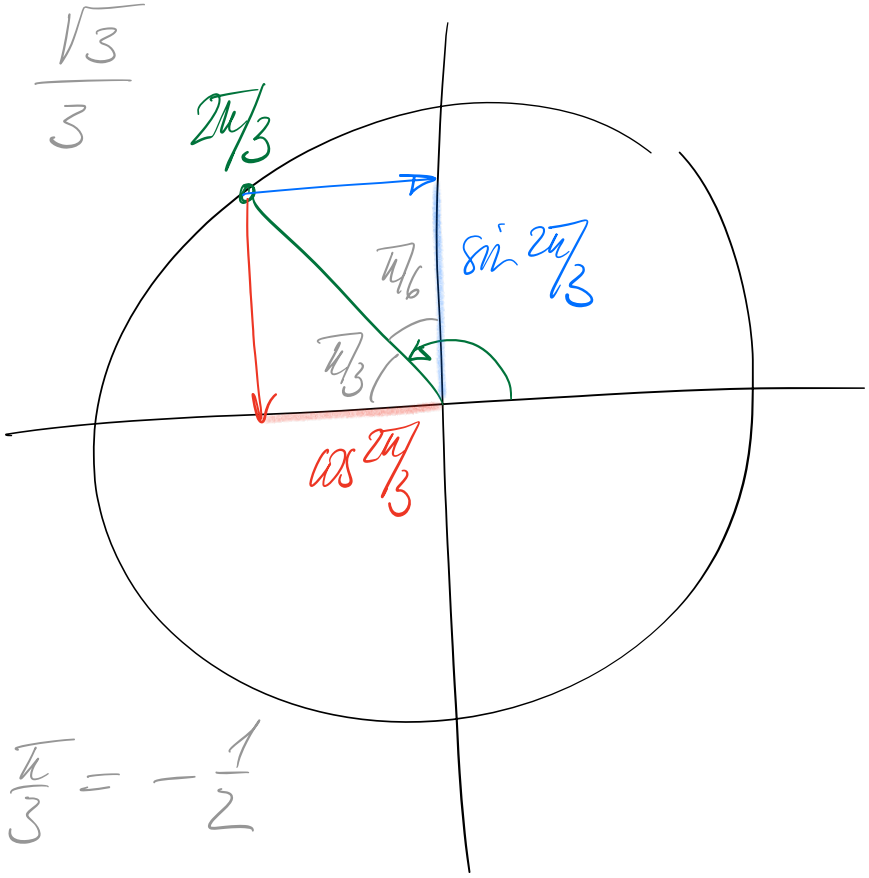


$$\sin\frac{\pi}{6} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sin\frac{\pi}{3} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan\frac{\pi}{3} = \sqrt{3}$$



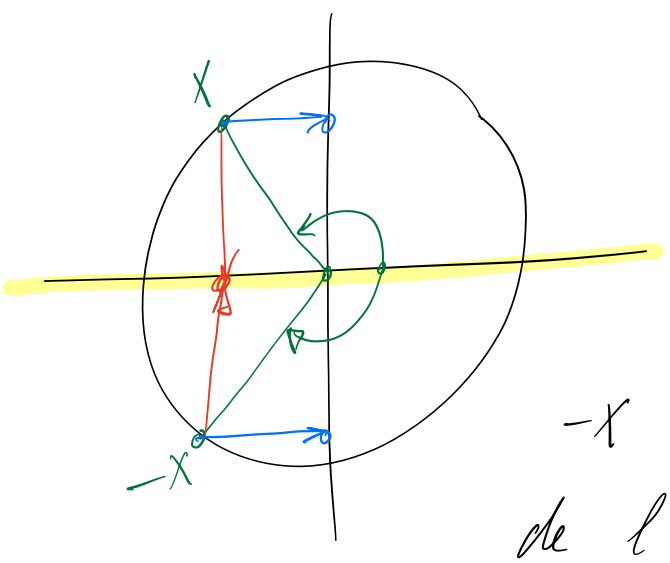
On en déduit :

$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Ce n'est qu'un exemple parmi bien d'autres.

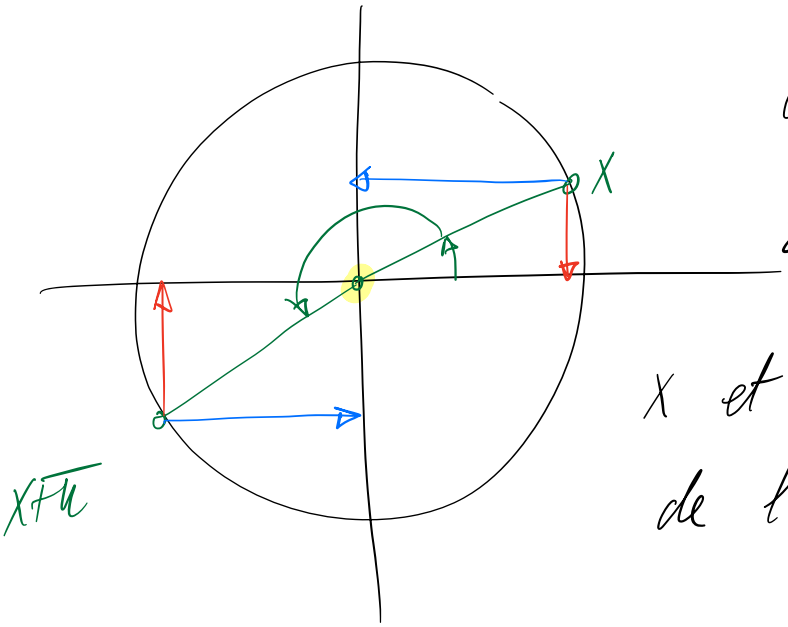
Quelques relations :



$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$

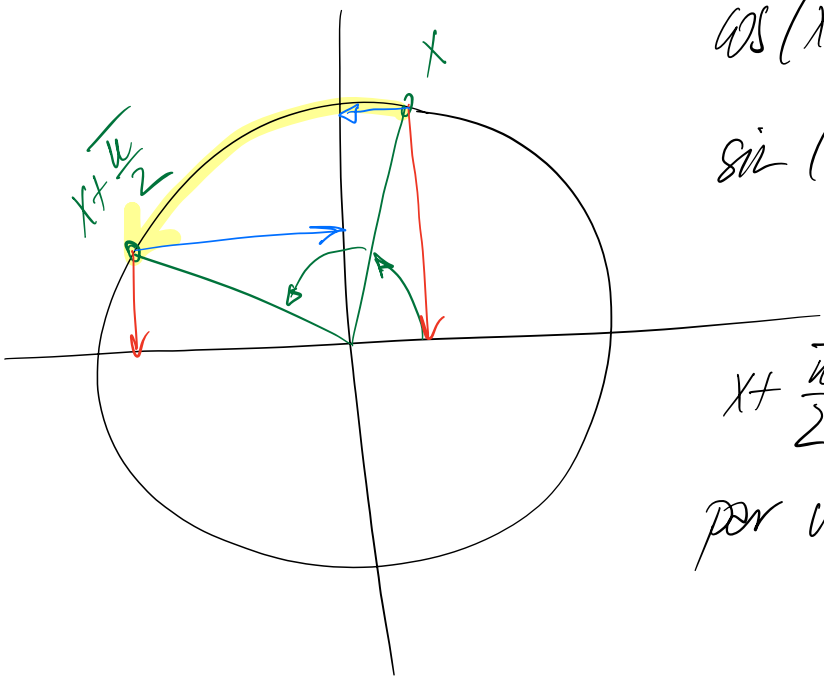
$-x$ et x sont image l'un de l'autre par une symétrie axiale.



$\cos(x+\pi) = -\cos x$

$\sin(x+\pi) = -\sin x$

x et $x+\pi$ sont image l'un de l'autre par une symétrie centrale.



$\cos\left(x+\frac{\pi}{2}\right) = -\sin x$

$\sin\left(x+\frac{\pi}{2}\right) = \cos x$

$x+\frac{\pi}{2}$ est l'image de x par une rotation de $\frac{\pi}{2}$