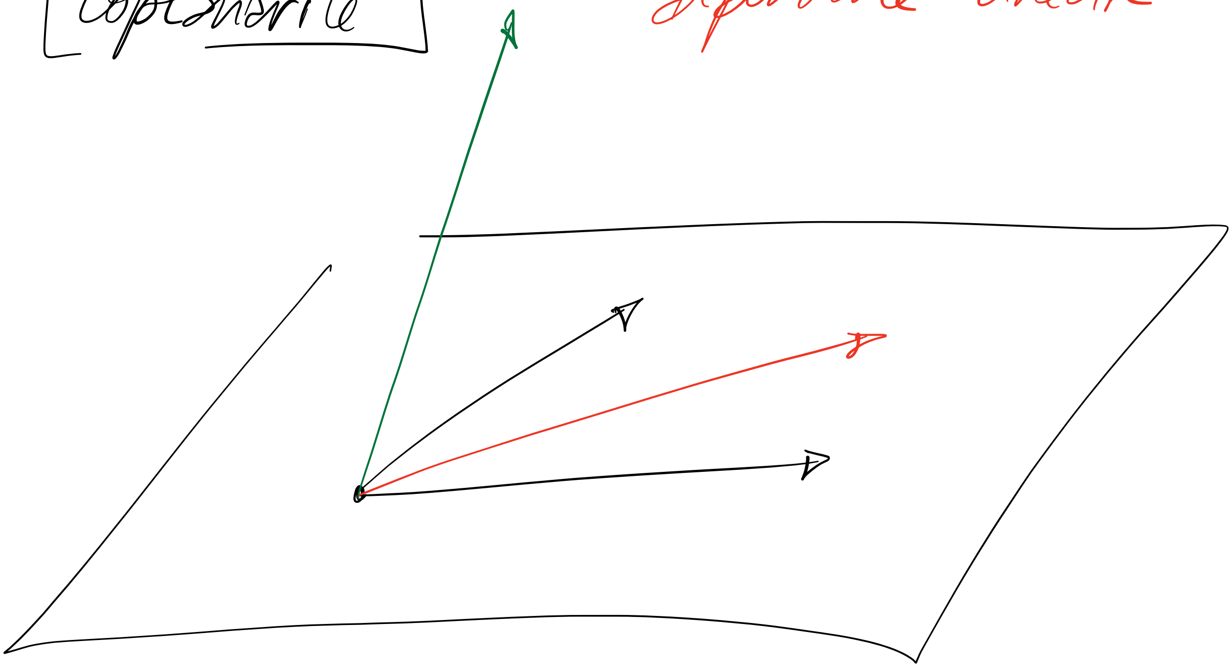
Coplanarité

Dépendance linéaire



Rappel: $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ et $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ sont colinéaires

$$\Leftrightarrow \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0 \Leftrightarrow a_1 b_2 - a_2 b_1 = 0$$

Prop. $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ et $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ sont coplanaires

si et seulement si

$$\det \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0$$

$$\Leftrightarrow \begin{array}{c} (-1)^{1+1} \\ (-1)^{2+1} \\ (-1)^{3+1} \end{array} \begin{vmatrix} a_1^{\oplus} & b_1 & c_1 \\ a_2^{\ominus} & b_2 & c_2 \\ a_3^{\oplus} & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Leftrightarrow a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$a_1 b_2 c_3 - a_1 b_3 c_2 - a_2 b_1 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1$$

$$\Leftrightarrow a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) = 0$$

$\boxed{\text{Règle de Sarrus}}$ Calcul du déterminant

$$\begin{array}{ccccc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}$$

$$- a_3 b_2 c_1 - b_3 c_2 a_1 - c_3 a_2 b_1$$

$$+ a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3$$

__Exemple__ : $\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}3\\9\\13\end{pmatrix}$ coplanaires ?

$4\begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}-1\\1\\1\end{pmatrix}$ Oui

$$\begin{matrix}1 & -1 & 3 & 1 & -1\\ 2 & 1 & 9 & 2 & 1\\ 3 & 1 & 13 & 3 & 1\end{matrix} \longrightarrow \begin{aligned}&13-27+6\\ &-9-9+26 =\\ &45-45=0\end{aligned}$$

1.3.3

$$\vec{a} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$\vec{x} = k \cdot \vec{a}$ $\quad$ $\vec{x}$ colinéaire à $\vec{a}$

$$\vec{x} = k \cdot \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

Équation vectorielle

$$\boxed{k\begin{pmatrix} 7 \\ -2 \end{pmatrix} + \lambda \cdot \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}}$$

$$\vec{x} + \lambda \cdot \vec{b} = \vec{c}$$

$$\begin{cases} 7k - 3\lambda = 0 \\ -2k + 5\lambda = 5 \end{cases}$$

1.3.6

$$\begin{vmatrix} x_1 & 1 & 0 \\ x_2 & -3 & 8 \\ x_3 & 2 & -5 \end{vmatrix} = 0$$

$$x_1(15-16) - x_2(-5-0) + x_3(8-0) = 0$$

$$-x_1 + 5x_2 + 8x_3 = 0$$

$$\boxed{x_1 - 5x_2 - 8x_3 = 0} \quad L_1$$

$$\begin{vmatrix} x_1 & 35 & -2 \\ x_2 & 14 & -1 \\ x_3 & -10 & 0 \end{vmatrix} = 0$$

$$x_1(-10) - x_2(-20) + x_3(-35+28) = 0$$

$$-10x_1 + 20x_2 - 7x_3 = 0$$

$$\boxed{10x_1 - 20x_2 + 7x_3 = 0} \quad L_2$$

$$\begin{cases} x_1 = 5x_2 + 8x_3 \\ 30x_2 + 87x_3 = 0 \\ \qquad x_3 = t \end{cases} \qquad L_2 \leftarrow L_2 - 10L_1$$

$$\begin{cases} x_1 = 5x_2 + 8x_3 \\ x_2 = -\frac{87}{30}t \\ x_3 = t \end{cases} \qquad \begin{cases} x_1 = -\frac{\cancel{5}\cdot 87}{\cancel{30}_{6}}t + 8t \\ x_2 = -\frac{87}{30}t \\ x_3 = t \end{cases}$$

$$x_1 = \frac{48-87}{6}\cdot t = \frac{-39}{6}\cdot t = \frac{-13}{2}\cdot t = \frac{-65}{10}\cdot t$$

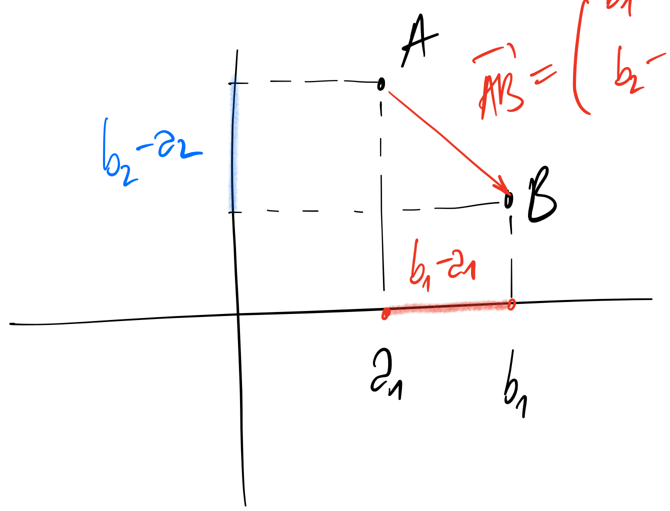
$$x_2 = -\frac{29}{10}\cdot t = -\frac{29}{10}\cdot t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} -\frac{65}{20} \\ -\frac{29}{20} \\ 1 \end{pmatrix}$$

$A(a_1; a_2; a_3)$

$B(b_1; b_2; b_3)$

$$\vec{AB} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$$



$$\vec{AB} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$$