Algèbre

→ 29/10/2026

2.1.1 à 2.1.4 « en boucle »

$(a+b)^2 = (a+b)(a+b) = a^2+ab+ba+b^2 = a^2+2ab+b^2$

$(a-b)^2 = (a-b)(a-b) = a^2-ab-ba+b^2 = a^2-2ab+b^2$

$(a+b)(a-b) = a^2-ab+ba-b^2 = a^2-b^2$

$(a+b)^3 = (a^2+2ab+b^2)(a+b) = a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$(a-b)^3 = (a^2-2ab+b^2)(a-b)$

$= a^3+3a^2b+3ab^2+b^3$

$= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

$= a^3-3a^2b+3ab^2-b^3$

$(a+b)^2 = a^2+2ab+b^2$ } 1 2 1

$(a-b)^2 = a^2-2ab+b^2$

$(a+b)(a-b) = a^2-b^2$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ } 1 3 3 1

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Exemplo: $(\underbrace{3x}_{a} - \underbrace{4y}_{b})^2 = (3x)^2 - 2\cdot(3x)(4y)+(4y)^2$

$= 9x^2 - 24xy + 16y^2$

2.1.1

$a(b+c) = ab + ac$

a) $3 + xz + y^2$

b) $3 - (xz + y^2) = 3 + (-1)(xz + y^2)$

$= 3 - xz - y^2$

c) $3(xz + y^2) = 3xz + 3y^2$

d)
$$\begin{array}{r} 2a + b - c \\ + \quad 3a - b + c \\ \hline 5a \quad 0b \quad 0c \\ \qquad\quad \underbrace{\qquad} \\ \qquad\quad 0 \end{array}$$

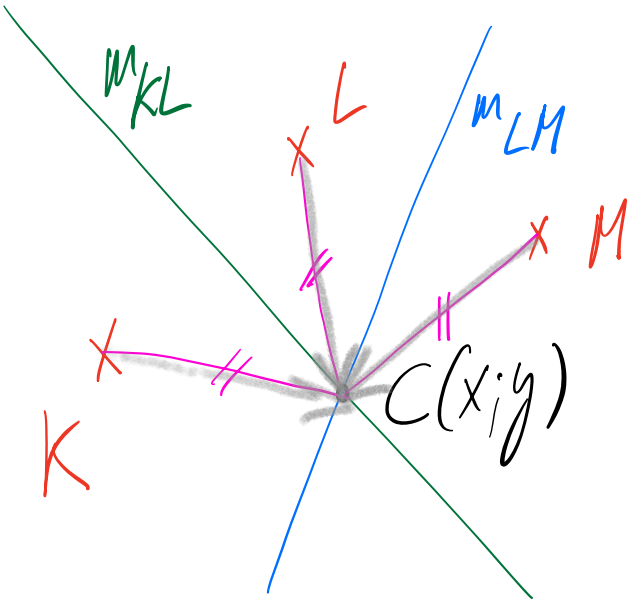
$\Rightarrow (2a + b - c) + (3a - b + c) = 5a$

$2a + b - c + 3a - b + c$

e) $(2a + b - c) - (3a - b + c) =$

$2a + b - c - 3a + b - c = \boxed{-a + 2b - 2c}$

1.4.7 $K(-3;6)$ $L(9;-10)$ $M(-5;4)$



$$\|\vec{MC}\| = \|\vec{LC}\|$$

$$\left\|\begin{pmatrix} x+5 \\ y-4 \end{pmatrix}\right\|^2 = \left\|\begin{pmatrix} x-9 \\ y+10 \end{pmatrix}\right\|^2$$

$$\|\vec{KC}\| = \|\vec{LC}\|$$

$$\left\|\begin{pmatrix} x+3 \\ y-6 \end{pmatrix}\right\|^2 = \left\|\begin{pmatrix} x-9 \\ y+10 \end{pmatrix}\right\|^2$$

$$\Rightarrow (x+5)^2+(y-4)^2 = (x-9)^2+(y+10)^2$$

$$x^2+10x+25+y^2-8y+16 = x^2-18x+81+y^2+20y+100$$

$$28x - 28y = 140$$

$$\boxed{x-y=5}$$

$$\Rightarrow (x+3)^2+(y-6)^2 = (x-9)^2+(y+10)^2$$

$$x^2+6x+9+y^2-12y+36 = x^2-18x+81+y^2+20y+100$$

$$24x - 32y = 136$$

$3x - 4y = 17$

A résoudre : $x = y + 5$

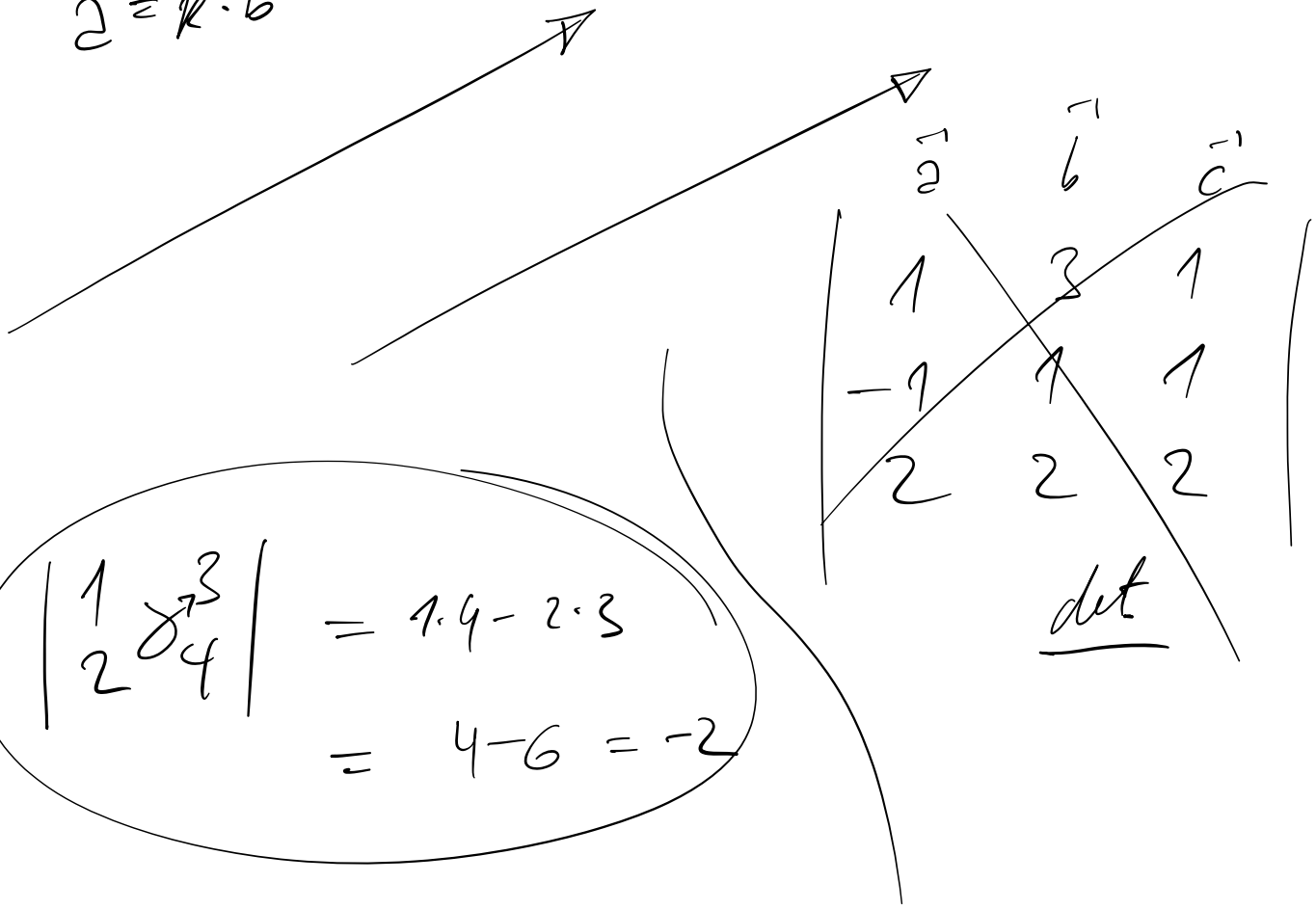
$3(y+5) - 4y = 17$

$-y = 2$

$y = -2 \qquad x = 3$

$\Rightarrow$ Le centre est $(3 ; -2)$

$\vec{a} = k \cdot \vec{b}$



$$\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3$$

$$= 4 - 6 = -2$$

$$k\begin{pmatrix}7\\-2\end{pmatrix}+\lambda\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}0\\5\end{pmatrix}$$

$$\begin{cases} 7k-3\lambda=0 & \cdot 5 \\ -2k+5\lambda=5 & \cdot 3 \end{cases}$$

$$35k-15\lambda=0$$

$$-6k+15\lambda=15$$

$$15\lambda=35k$$

$$\lambda=\frac{35\cdot 15}{15\cdot 29}=\frac{35}{29}$$

$$29k=15$$

$$k=\frac{15}{29}$$

$$\frac{15}{29}\cdot\begin{pmatrix}\frac{7}{1}\\-\frac{2}{1}\end{pmatrix}=\begin{pmatrix}\frac{15}{29}\cdot\frac{7}{1}\\ \frac{15}{29}\cdot\frac{(-2)}{1}\end{pmatrix}=\begin{pmatrix}\frac{15\cdot 7}{29\cdot 1}\\ \frac{15\cdot(-2)}{29\cdot 1}\end{pmatrix}$$

$$\vec{x}=\begin{pmatrix}105/29\\-30/29\end{pmatrix}$$

1.3.12

A, B et C alignés $\Leftrightarrow \overrightarrow{AB} = x \cdot \overrightarrow{AC}$

plan $\Leftrightarrow \det(\overrightarrow{AB}; \overrightarrow{AC}) = 0$

a) $A(1;2)$ $B(-3;3)$ $C(k;1)$

$$\overrightarrow{AB} = \begin{pmatrix} -3-1 \\ 3-2 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} k-1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 1 \end{pmatrix} = x \cdot \begin{pmatrix} k-1 \\ -1 \end{pmatrix} \Leftrightarrow \begin{matrix} -4 = x(k-1) \\ 1 = -x \end{matrix}$$

$\Rightarrow x = -1$

$\Rightarrow -4 = -(k-1)$

$\Rightarrow k-1 = 4$

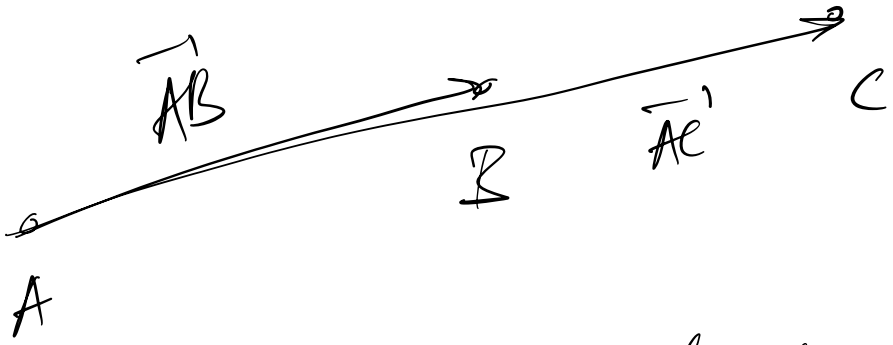
$\Rightarrow \boxed{k = 5}$

$$\begin{vmatrix} -4 & k-1 \\ 1 & -1 \end{vmatrix} = 0$$

$\Leftrightarrow 4 - (k-1) = 0$

$\Leftrightarrow k-1 = 4$

$\Leftrightarrow k = 5$



A, B et C alignés $\Leftrightarrow$ $\overrightarrow{AB} = k \cdot \overrightarrow{AC}$

↑ système

