$$f(x) = \frac{x^3}{x^2-1} = \frac{x^3}{(x+1)(x-1)}$$

① $D_f = \mathbb{R} - \{\pm 1\}$

$$f(x) = 0 \iff x^3 = 0 \iff x = 0$$

	-1		0		1	
$-$	‖	$+$	0	$-$	‖	$+$

$$f(2) = \frac{8}{4-1} > 0$$

② $$\lim_{x \to -1} f(x) = \left\langle\!\left\langle \frac{-1^3}{0} \right\rangle\!\right\rangle = \infty$$

$$\boxed{\Rightarrow \text{A.V. en } x = -1}$$

$$\lim_{x \to 1} f(x) = \left\langle\!\left\langle \frac{1^3}{0} \right\rangle\!\right\rangle = \infty$$

$$\boxed{\Rightarrow \text{A.V. en } x = 1}$$

$$\deg(x^3) - \deg(x^2-1) = 3-2 = 1$$

$\Rightarrow$ On a une A.O. et pas d'A.H.

$$\begin{array}{l|l} x^3 & x^2-1 \\ \hline x^3 - x & x \\ \hline \quad x & \end{array}$$

$$x^3 = x\cdot(x^2-1) + x$$

$$\frac{x^3}{x^2-1} = x + \boxed{\frac{x}{x^2-1}}$$

$$\delta(x) = \frac{x}{(x+1)(x-1)}$$

On a une A.O. en

$$\boxed{y = x = 1\cdot x + 0}$$

$$\delta(2) = \frac{2}{4-1} > 0$$

Position : signe de δ

x		-1		0		1	
$\delta(x)$	$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$
	dessous						dessus

③ $$f'(x) = \left(\frac{x^3}{x^2-1}\right)' = \frac{(x^3)'(x^2-1) - x^3\,(x^2-1)'}{(x^2-1)^2}$$

$$= \frac{3x^2(x^2-1) - x^3\cdot 2x}{(x^2-1)^2}$$

$$= \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} = \frac{x^4-3x^2}{(x^2-1)^2}$$

$$f'(2) = \frac{4\cdot 1}{9} > 0$$

$$= \frac{x^2(x^2-3)}{(x^2-1)^2}$$

$$x^2-3=0 \iff x^2=3$$

$$f'(x)=0 \iff x=0 \text{ ou } x=\pm\sqrt{3}$$

x		$-1{,}73$		-1		0		1		$1{,}73$	
$f'(x)$	$+$	0	$-$	$\Vert$	$-$	0	$-$	$\Vert$	$-$	0	$+$
f	$\nearrow$	MAX	$\searrow$	$\Vert$	$\searrow$	PALIER	$\searrow$	$\Vert$	$\searrow$	MIN	$\nearrow$

Maximum : $\left(-\sqrt{3}\,;\, f(-\sqrt{3})\right) \simeq (-1{,}7\,;\,-2{,}6)$

Minimum : $\left(\sqrt{3}\,;\, f(\sqrt{3})\right) \simeq (1{,}7\,;\,2{,}6)$

Palier : $\left(0\,;\, f(0)\right) = (0\,;\,0)$

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