$$f(x) = \frac{x^2}{9-x^2} = \frac{x \cdot x}{(3-x)^1(3+x)^1} = \frac{x^2}{(3-x)^1(3-x)^1}$$

① $D_f = \mathbb{R} - \{\pm 3\}$

Zeros : $f(x) = 0 \Leftrightarrow \dfrac{x^2}{9-x^2} = 0 \Leftrightarrow x^2 = 0$ et $x \neq \pm 3$

$$\Leftrightarrow \boxed{x = 0}$$

Signe :

x		-3		0		3	
$f(x)$	$-$	$\|$	$+$	0	$+$	$\|$	$-$

$$f(1) = \frac{1}{9-1} = \frac{1}{8} > 0$$

② $\boxed{\text{A.V.}}$

$$\lim_{x \to -3} f(x) = « \frac{(-3)^2}{9-(-3)^2} » = « \frac{9}{9-9} » = « \frac{9}{0} » = \infty$$

$$\Rightarrow \boxed{\text{A.V. en } x = -3}$$

$$\lim_{x \to 3} f(x) = « \frac{3^2}{9-3^2} » = « \frac{9}{9-9} » = « \frac{9}{0} » = \infty$$

$$\Rightarrow \boxed{\text{A.V. en } x = 3}$$

$$f(x) = \frac{x^2}{9 - x^2} \qquad \underbrace{\deg(x^2) = 2 = \deg(9 - x^2)}_{\Rightarrow \text{ A.H.}}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2}{9 - x^2} = \lim_{x \to \infty} \frac{x^2}{-x^2} = -1$$

$$\Rightarrow \boxed{\text{A.H. en } y = -1}$$

Position de f relativement à $y = -1$:

$$f(x) - (-1) = \frac{x^2}{9 - x^2} + 1 = \frac{x^2}{9 - x^2} + \frac{9 - x^2}{9 - x^2}$$

$$= \frac{x^2 + 9 - x^2}{9 - x^2} = \frac{9}{9 - x^2}$$

	-3		3	
$-$	||	$+$	||	$-$
dessous				dessous

③ $$f'(x) = \frac{(x^2)'(9-x^2) - x^2\cdot(9-x^2)'}{(9-x^2)^2}$$

$$= \frac{2x(9-x^2) - x^2\cdot(-2x)}{(9-x^2)^2}$$

$$= \frac{18x - x^3 + 2x^3}{(9-x^2)^2} = \frac{x^3+18x}{(9-x^2)^2}$$

$$= \frac{x(x^2+18)}{(9-x^2)^2}$$

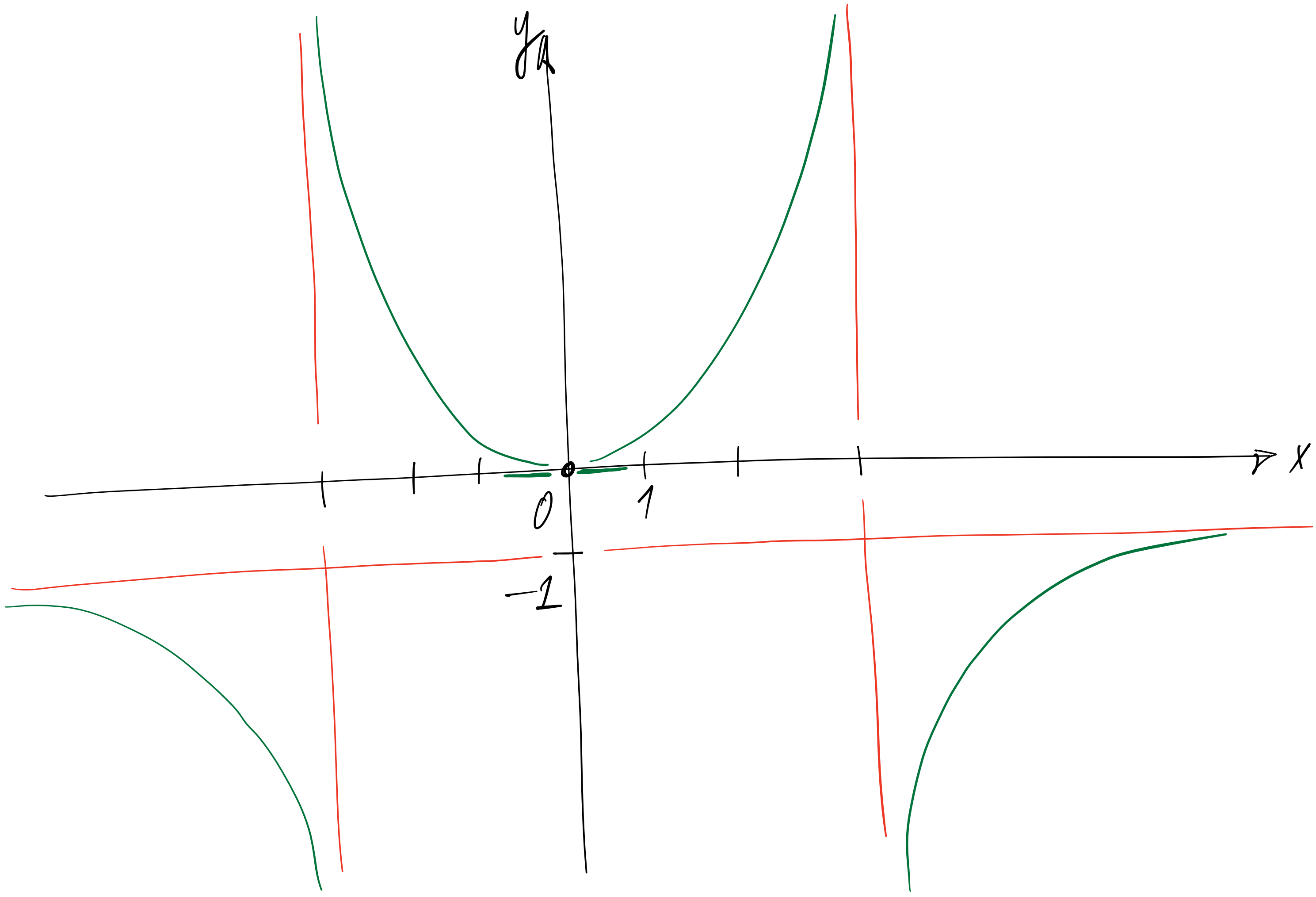
$f'(x) = 0 \Leftrightarrow x = 0$

	-3		0		3	
$-$	$\Vert$	$-$	0	$+$	$\Vert$	$+$
$\searrow$		$\searrow$	MIN	$\nearrow$		$\nearrow$

MIN

$(0;0)$

$$f'(1) = \frac{1\cdot 19}{8^2} > 0$$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