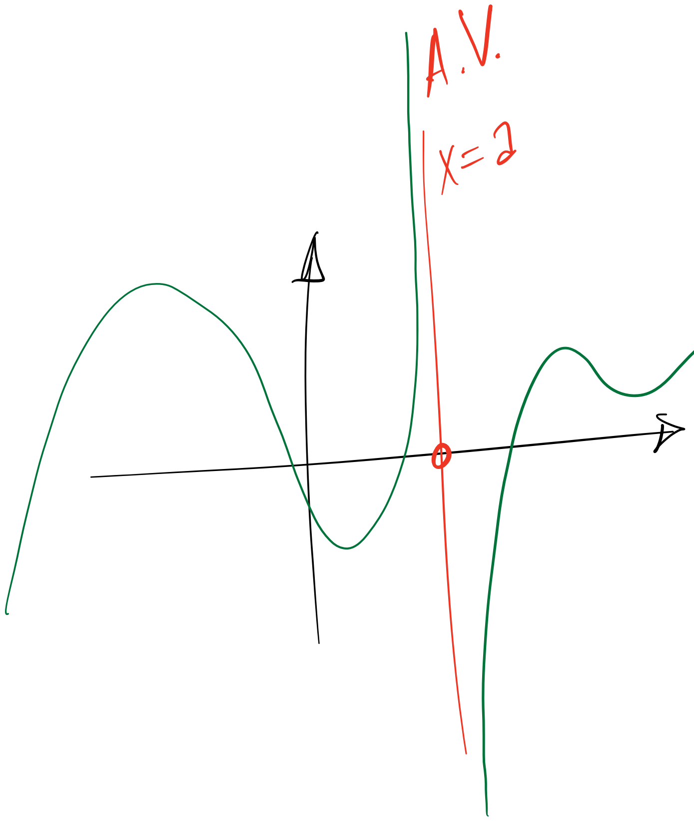
ASYMPTOTES

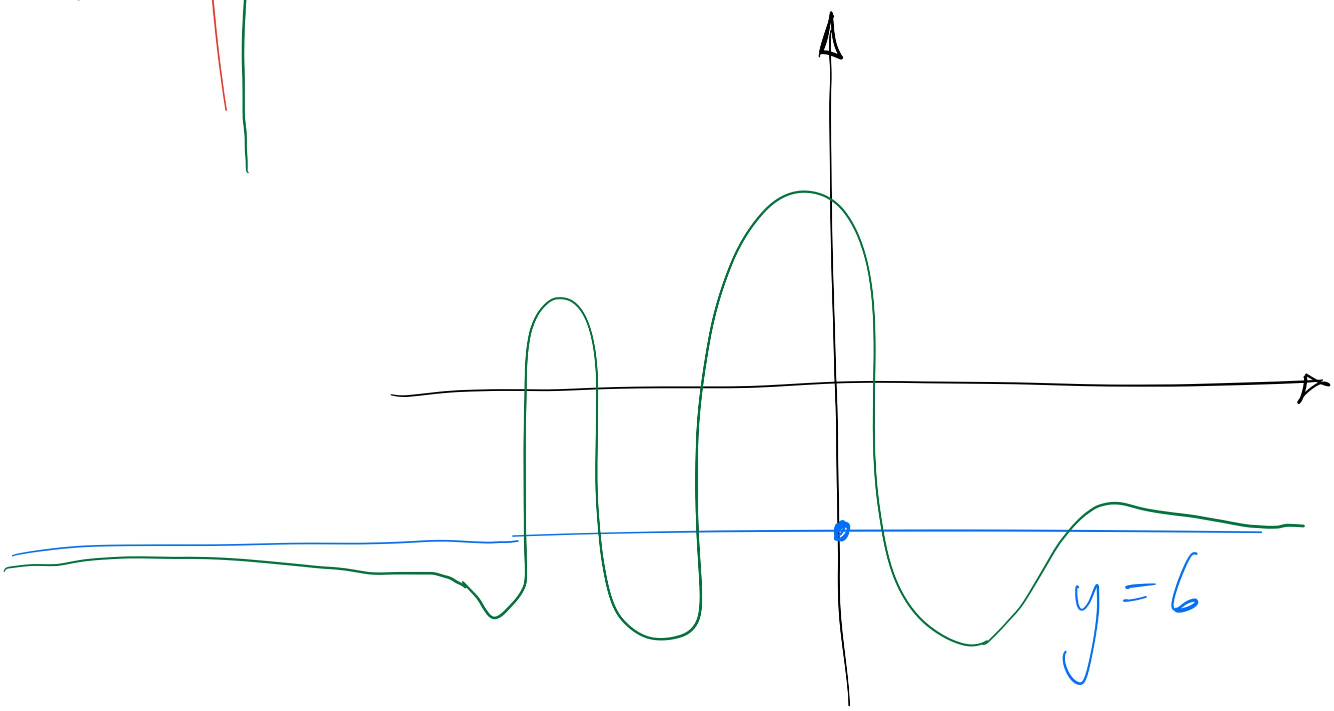
VERTICALES

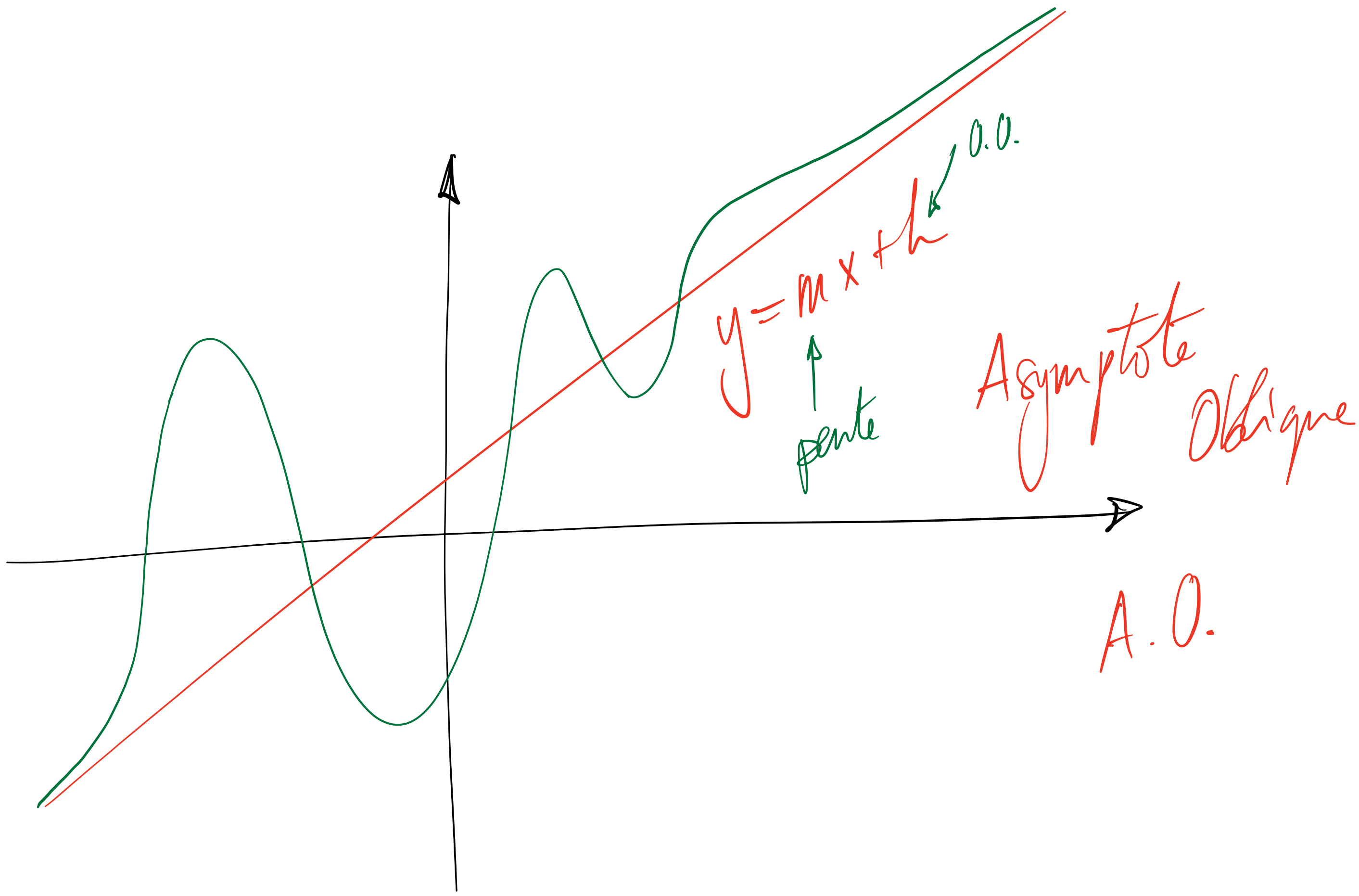
HORIZONTALES

OBLIQUES

POSITION







Exemple :

$$f(x) = \frac{x^2+2x+1}{x-3}$$

Asymptotes :

$\boxed{\text{A.V.}}$ ✓ $\lim\limits_{x \to 3} \dfrac{x^2+2x+1}{x-3} = « \dfrac{9+6+1}{0} » = « \dfrac{16}{0} »$

$= \infty$

$\Rightarrow$ A.V. en $x = 3$

exclusif

~~A.H.~~ ?

A.O. ?

$$\frac{x^2+2x+1}{x-3}$$

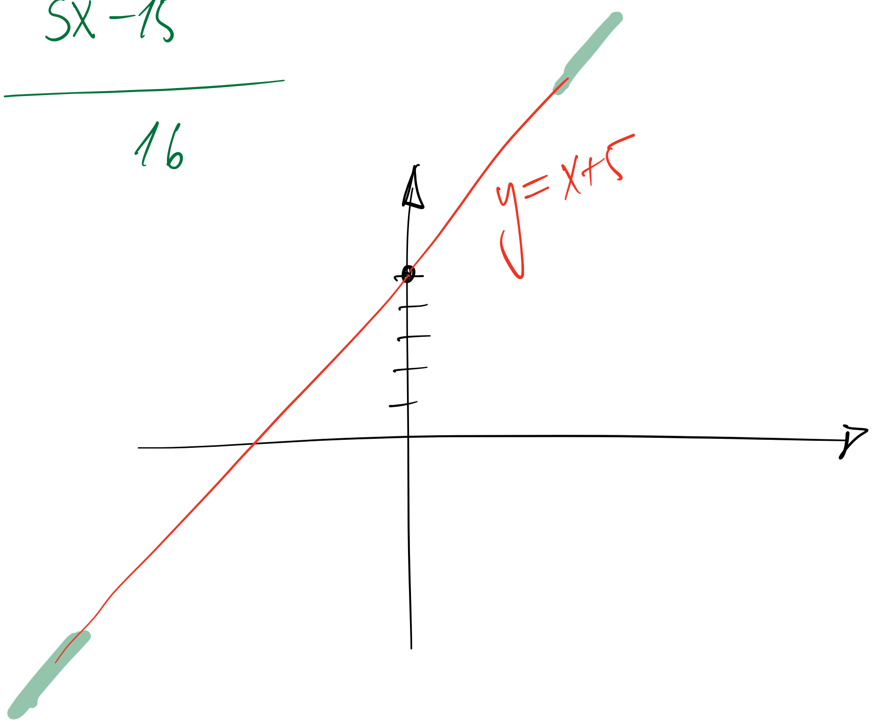
$$\frac{\text{degré : } 2}{\text{degré : } 1}$$

différence : 2-1 = 1

haut-bas

$$\begin{array}{r|l} x^2+2x+1 & x-3 \\ \hline x^2-3x & x+5 \\ \hline 5x+1 & \\ 5x-15 & \\ \hline 16 & \end{array}$$

A.O. en $y = x+5$



A.H. / A.O. d'une fraction de polynômes

$$f(x) = \frac{P(x)}{Q(x)}$$

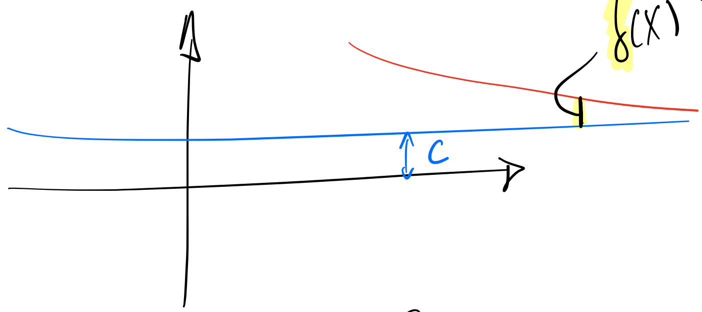
$\deg P(x) = \deg Q(x)$ $\Leftrightarrow \boxed{\deg P(x) - \deg Q(x) = 0}$	A.H. en $y = \lim\limits_{x \to \infty} \frac{P(x)}{Q(x)} = c$ $\frac{2x^3-1}{3x^3+2x+1}$
$\boxed{\deg P(x) - \deg Q(x) = 1}$ $\frac{x^3+1}{x^2-2}$	A.O. en $y = mx+h$ $\begin{array}{l\|l} P(x) & Q(x) \\ \hline & m \cdot x + h \end{array}$ division euclidienne
$\deg P(x) < \deg Q(x)$ $\Leftrightarrow \boxed{\deg P(x) - \deg Q(x) < 0}$	A.H. en $y = 0$ $\frac{x+1}{x^3-2x^2+1}$
$\deg P(x) > \deg Q(x)$ $\Leftrightarrow \boxed{\deg P(x) - \deg Q(x) > 1}$	~~A.H.~~ ~~A.O.~~ $\frac{x^4+1}{x}$

Position du graphe de f relativement à une A.H.

$$\lim_{x\to\infty} f(x) = c \Rightarrow \text{A.H. en } y = c$$

delta minuscule: δ

$$\delta(x) = f(x) - c$$



$$f(x) = \frac{P(x)}{Q(x)}$$

Le signe de $\delta(x)$ renseigne sur la position :

si $\delta(x) > 0$ f est en dessus de l'A.H.

si $\delta(x) < 0$ f est en dessous de l'A.H.

Exemple: $$f(x) = \frac{3x^2-2x+5}{-2x^2+x+1}$$ Δ degrés: 0

$$\lim_{x\to\infty} f(x) = -\frac{3}{2} \Rightarrow \text{A.H. en } y = -\frac{3}{2}$$

$$\delta(x) = \frac{3x^2-2x+5}{-2x^2+x+1} - \left(-\frac{3}{2}\right) = \frac{2(3x^2-2x+5)+3(-2x^2+x+1)}{2(-2x^2+x+1)}$$

$$= \frac{\cancel{6x^2} - 4x + 10 - \cancel{6x^2} + 3x + 3}{2(-2x^2+x+1)} = \frac{-x+13}{-4x^2+2x+2}$$

$$= \frac{x-13}{4x^2-2x-2} = f(x)$$

Signe à trouver

Zero: $x = 13$

À exclure:

$$x = \frac{2 \pm \sqrt{4+32}}{8} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

	$-0{,}5$		1		13	
$-$	$\Vert$	$+$	$\Vert$	$-$	0	$+$