TE de Noël

CRM 30 chap. 2

pp. 66 et suivantes

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MERCREDI 11/12/2024

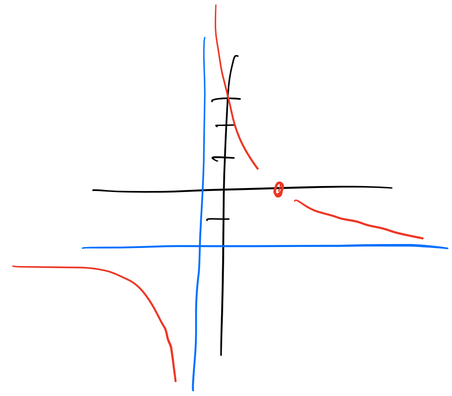
p. 67 CRM 30 : exercice 37

fraction de polynômes

ÉTUDE D'UNE FONCTION :

① ED_f / Zéros / Signe

② $\boxed{\text{ASYMPTOTES}}$



Exemple: f $\boxed{\text{homographique}}$

Zéro: $x = 1,5$

$$f(x) = \frac{2 - 3x}{2x + 1}$$

A.H. en $y = -1,5$

A.V. en $x = -0,5$

$$f(x) = \frac{x^2 + x - 1}{x - 3}$$

$\lim_{x \to -0,5} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -1,5$

FRACTIONS DE POLYNÔMES :

① Factoriser $\dfrac{\text{haut}}{\text{bas}}$

$D_f = \mathbb{R} - \{3\}$

$$\frac{x^2+x-1}{x-3} = \frac{(x-0{,}618)(x+1{,}618)}{x-3}$$

x^2+x-1

$\Delta = 1^2 - 4 \cdot 1 \cdot (-1)$

$= 5$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$: $0{,}618$; $-1{,}618$

② A.V.

$$\lim_{x \to 3} \frac{(x-0{,}618)(x+1{,}618)}{(x-3)} = « \frac{2{,}4 \cdot 4{,}6}{0} » = \infty$$

$\boxed{\text{A.V. en } x = 3}$

③ A.H.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2+x-1}{x-3}$$

$$= \lim_{x \to \infty} \frac{x^2}{x} = \lim_{x \to \infty} x = \infty$$

~~A.H.~~

$$\boxed{\text{Etude de } f(x) = \frac{2-3x}{2x+1}}$$

$f(x) = 0$ si $2-3x=0$
(et que $2x+1 \neq 0$)

① $D_f = \mathbb{R} - \{-0{,}5\}$ car $2x+1=0 \Leftrightarrow x = -\frac{1}{2}$

Zéro : $2-3x=0 \Leftrightarrow 2=3x \Leftrightarrow x = \frac{2}{3}$

Signe :

x		$-0{,}5$		$0{,}67$	
	$-$	$\|\|$	$+$	0	$-$

$f(0) = 2$

② $\boxed{\text{A.V.}}$

$$\lim_{x \to -0{,}5} f(x) = \text{«} \frac{2-3(-0{,}5)}{2\cdot(-0{,}5)+1} \text{»} = \text{«} \frac{3{,}5}{0} \text{»}$$

$$= \infty$$

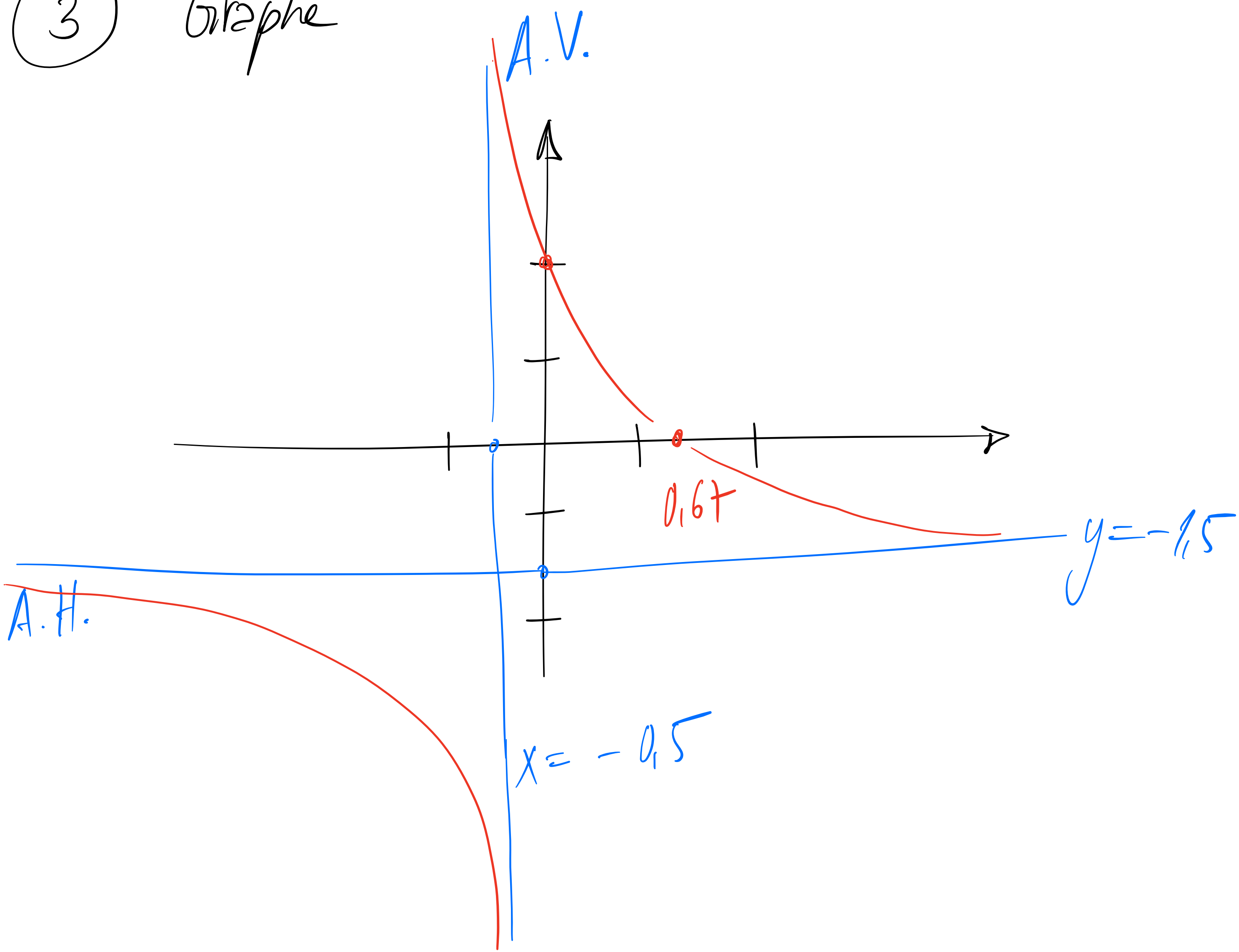
$\boxed{\text{A.V. en } x = -0{,}5}$

$\boxed{A.H.}$ $\lim\limits_{x \to \infty} f(x)$

$$\frac{2-3x}{2x+1} \xrightarrow{x\to\infty} \frac{-3x}{2x} = -\frac{3}{2} \xrightarrow{x\to\infty} -\frac{3}{2}$$

$\boxed{\text{A.H. en } y = -1{,}5}$

③ Graphe



$$(A - B)\cdot(A + B)$$

$$\frac{\sqrt{x+1}-2}{3-x}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} =$$

$$\frac{-\cancel{(B-A)}}{\cancel{(B-A)}} = \frac{A-B}{B-A} = -1$$

$$A^2 - 2^2$$

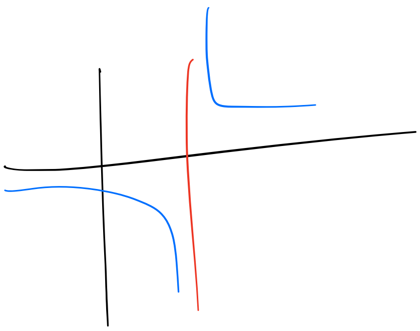
$$\frac{x+1-4}{(3-x)(\sqrt{x+1}+2)} = \frac{x-3}{(3-x)(\sqrt{x+1}+2)}$$

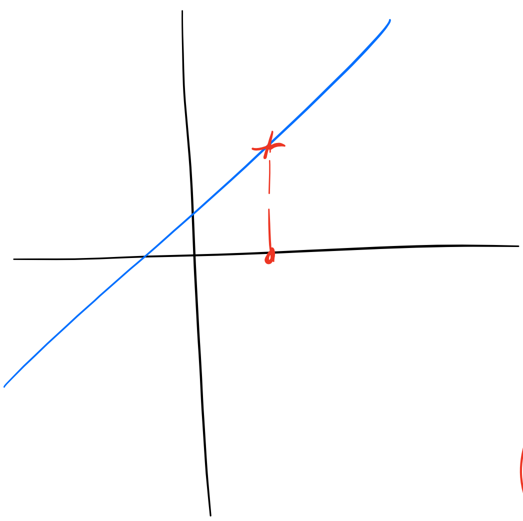
PASSAGE À LA LIMITE

$$= \frac{-1}{\sqrt{x+1}+2} \xrightarrow{x\to 3} \frac{-1}{\sqrt{3+1}+2}$$

$$= -\frac{1}{2+2}$$

$$= -\frac{1}{4}$$





$$\frac{x^2-1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} \quad \text{si } x \neq -1$$

$$g(-1) = -2$$

$$g(x) = \frac{x^2-1}{x+1} \quad \text{si } x \neq -1$$

g s'obtient en prolongeant f par continuité.

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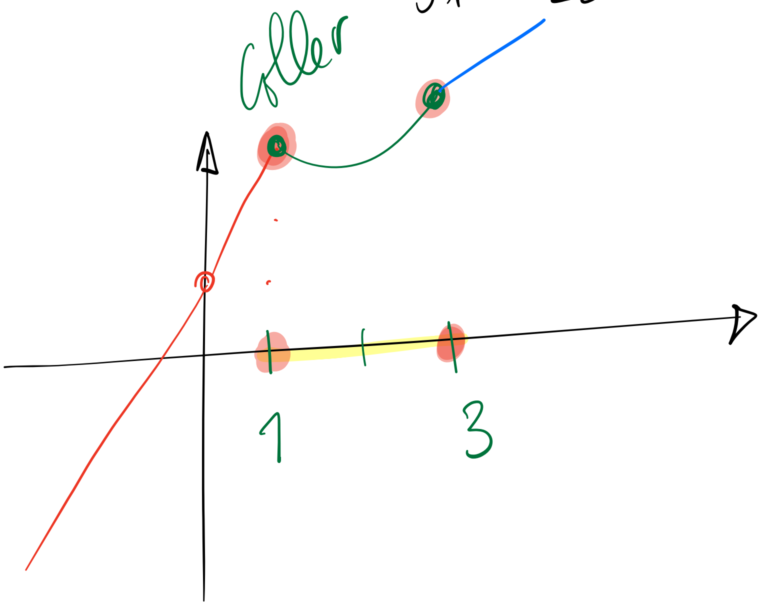
$$f(x) = \begin{cases} 2x+1 & \text{si } x \le 1 \\ ax^2+b & \text{si } 1 < x < 3 \\ 5x + 2a & \text{si } x \ge 3 \end{cases}$$



$\boxed{x=1}$

$2\cdot 1+1 = a\cdot 1^2+b$

$$\boxed{3 = a+b}$$

$x=3$

$a(3)^2+b = 5\cdot 3+2a$

$$\boxed{9a+b = 15+2a}$$

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1) $f(x) = \frac{2}{x-1}$

$2 = 0$ $\emptyset$

$x - 1 = 0 \Leftrightarrow x = 1$

$D_f = \mathbb{R} - \{1\}$

Pas de zeros

	1	
$-$	$\Vert$	$+$

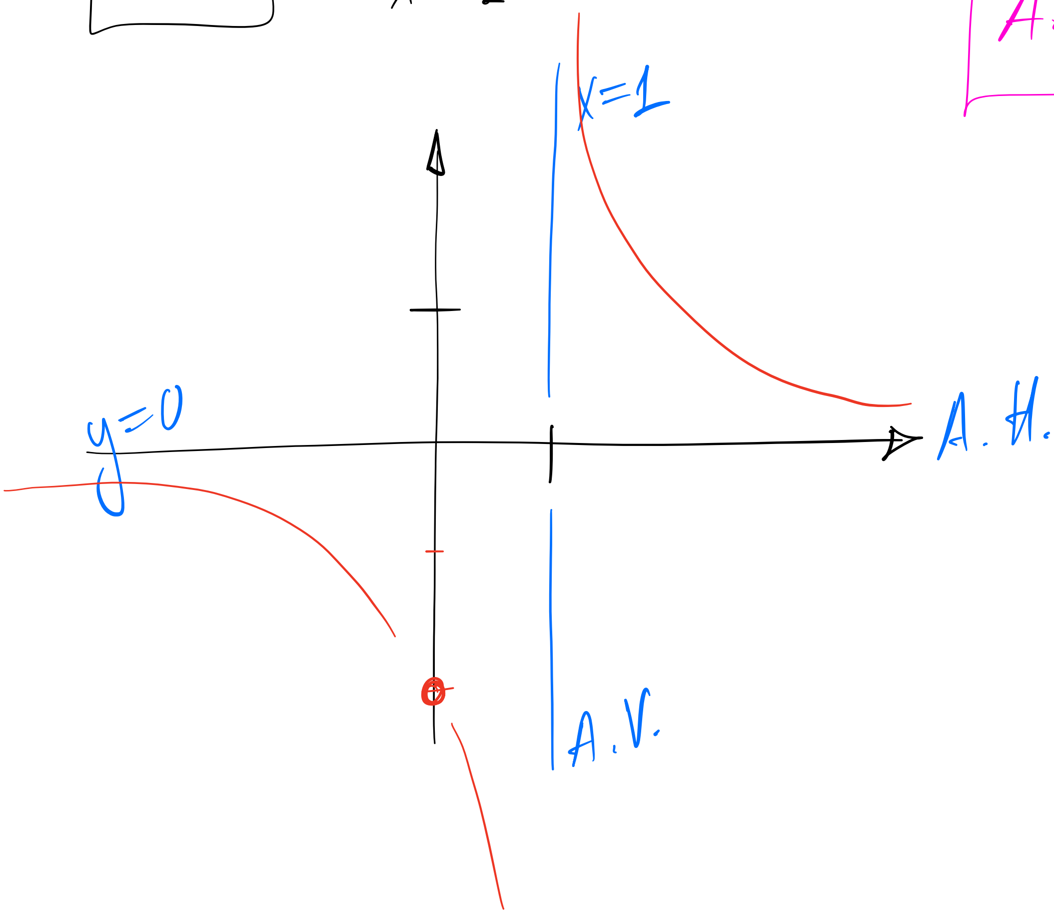
$f(0) = -2$

$\boxed{A.H.}$ $\lim\limits_{x \to \infty} \frac{2}{x-1} = \lim\limits_{x \to \infty} \frac{2}{x} = 0$

$\boxed{A.H. \text{ en } y = 0}$

$\boxed{A.V.}$ $\lim\limits_{x \to 1} \frac{2}{x-1} = « \frac{2}{0} » = \infty$

$\boxed{A.V. \text{ en } x = 1}$



$f(0) = -2$

2) $f(x) = \dfrac{x^2}{x^2-4} = \dfrac{x \cdot x}{(x+2)(x-2)}$

$D_f = \mathbb{R} - \{-2; 2\}$

Zéros: $x \cdot x = 0 \Leftrightarrow \boxed{x=0}$

$f(1) = \dfrac{1}{1-4} < 0$

$f(-1) = \dfrac{1}{1-4} < 0$

$f(3) = \dfrac{9}{9-4} > 0$

$f(-3) = \dfrac{9}{9-4} > 0$

Signe:

	-2		0		2	
$+$	$\|\|$	$-$	0	$-$	$\|\|$	$+$

$\boxed{\text{A.V.}}$ $\lim\limits_{x \to 2} f(x) = « \dfrac{2^2}{2^2-4} »$

$= « \dfrac{4}{0} » = \infty$

$\Rightarrow \boxed{\text{A.V. en } x = 2}$

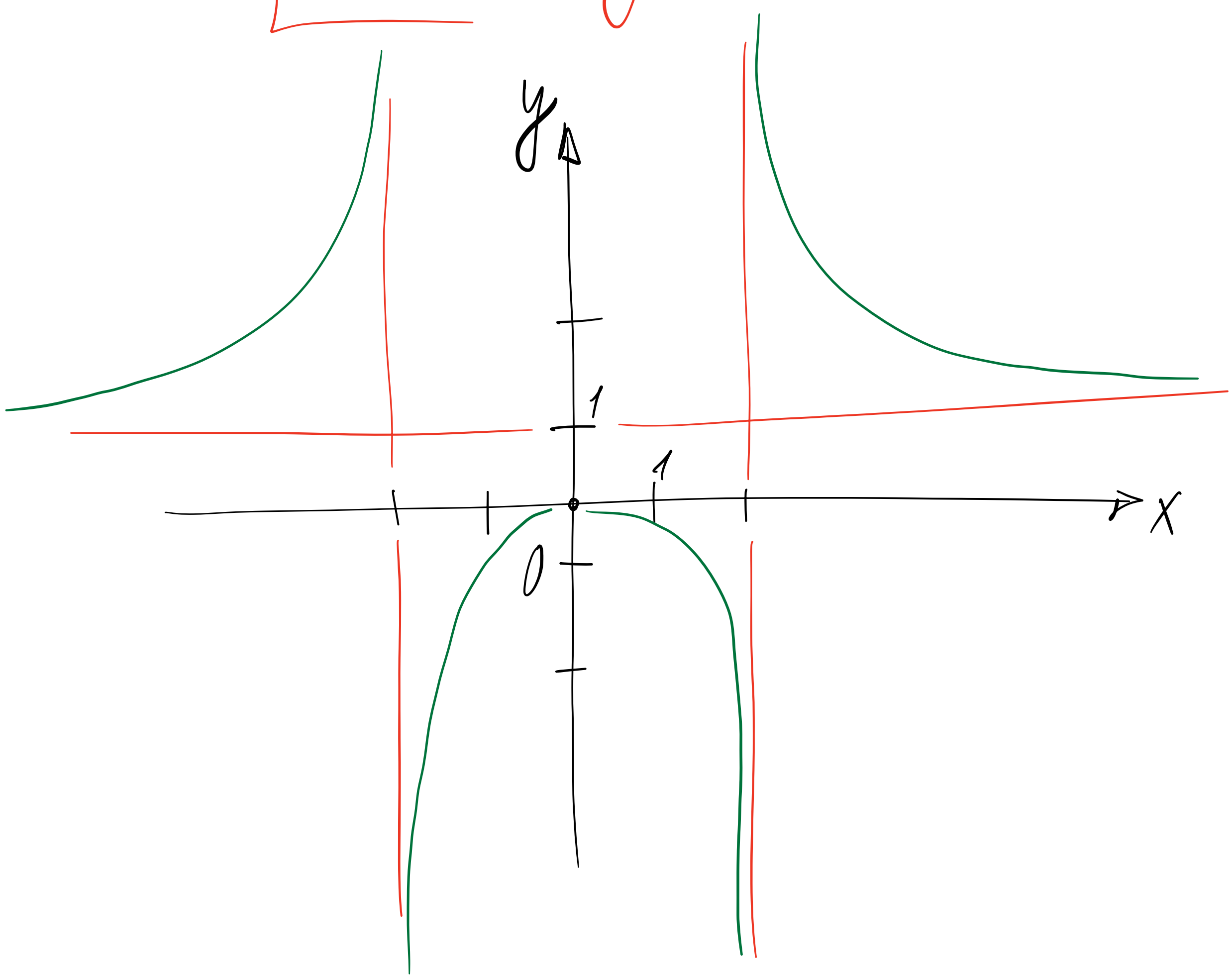
$\lim\limits_{x \to -2} f(x) = « \dfrac{(-2)^2}{(-2)^2-4} » = « \dfrac{4}{0} » = \infty$

$\Rightarrow \boxed{\text{A.V. en } x = -2}$

$\boxed{A.H.}$ $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2}{x^2-4}$

$= \lim\limits_{x\to\infty} \frac{x^2}{x^2} = 1$

$\Rightarrow$ $\boxed{\text{A.H. en } y = 1}$



3) $$f(x) = \frac{x^2-3x+4}{x^2-4x+3}$$

$$x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2}$$

$$x^2-4x+3=0 \Leftrightarrow (x-3)(x-1)=0 \Leftrightarrow \begin{array}{l} x=1 \checkmark \\ x=3 \checkmark \end{array}$$

$$\Rightarrow D_f = \mathbb{R} - \{1;3\}$$

$$x^2-3x+4=0 \Leftrightarrow x = \frac{3 \pm \sqrt{9-16}}{2} = \frac{3 \pm \cancel{\sqrt{-7}}}{2}$$

$\Rightarrow f$ n'a pas de zéro.

$$f(0) = \frac{4}{3} > 0$$

$$f(2) = \frac{2}{-1} < 0$$

Signe:

x		1		3	
	$+$	$\|\|$	$-$	$\|\|$	$+$

$\boxed{\text{A.H.}}$ $$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2}{x^2} = 1$$

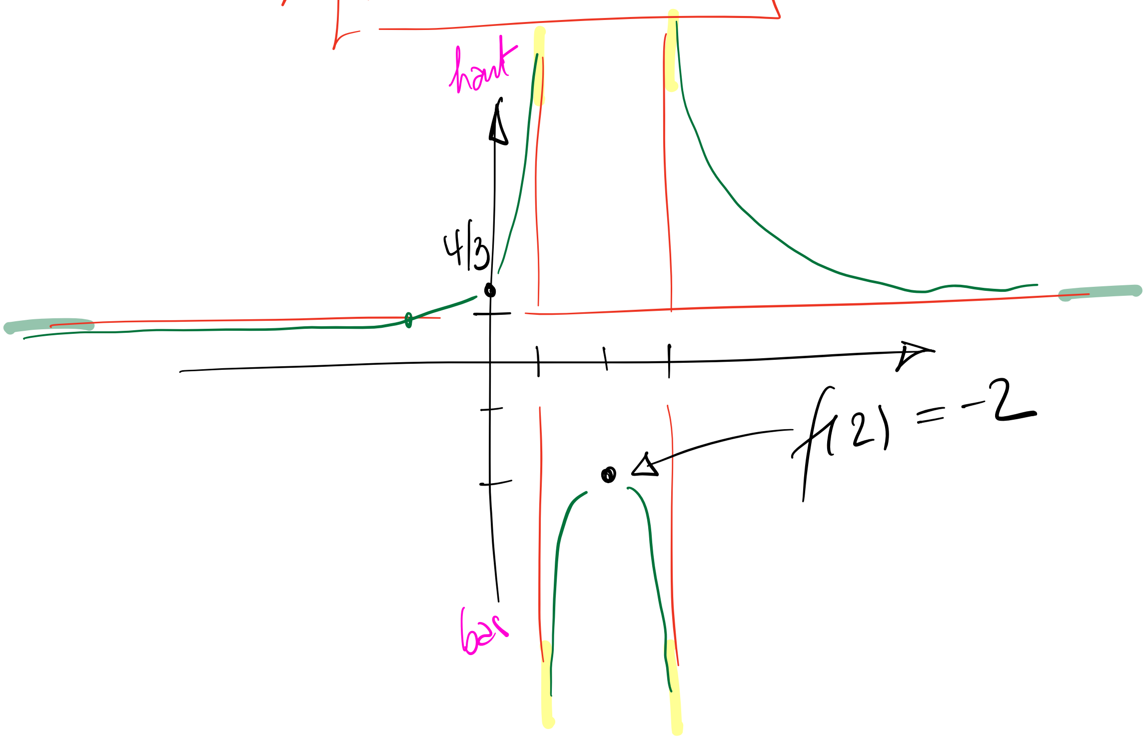
$\boxed{\text{A.H. en } y=1}$

$\boxed{\text{A.V.}}$ $$\lim_{x \to 1} f(x) = « \frac{2}{0} » = \infty$$

$\Rightarrow$ $\boxed{\text{A.V. en } x=1}$

$$\lim_{x \to 3} f(x) = « \frac{4}{0} » = \infty$$

$\Rightarrow$ $\boxed{\text{A.V. en } x=3}$



$2 \notin D_f$ 2 est un nombre à exclure

$\lim\limits_{x\to 2} f(x) = \infty \Rightarrow \boxed{A.V. \text{ en } x=2}$

$\lim\limits_{x\to 2} f(x) = 6 \Rightarrow$ ~~A.V.~~

TROU en $x=2 / y=6$

$\lim\limits_{x\to \infty} f(x) = c \Rightarrow \boxed{A.H. \text{ en } y=c}$

$\lim\limits_{x\to \infty} f(x) = \infty \Rightarrow$ ~~A.H.~~