$\boxed{2.8.10}$

e) $$f(x) = \frac{x^3}{x^2-4} = \frac{x^3}{(x+2)(x-2)}$$

① ED, zéros, tableau des signes

② Asymptotes A.V. | A.H. ou A.O. | S(x) pos. relative

tableau des signes $\rightarrow$ S(x)

③ Dérivée et croissance (tableau des signes de f')

④ Graphe

Cf. p. 91 CRM 30

$$f(x) = \frac{x^3}{(x+2)^1(x-2)^1}$$

① $ED = \mathbb{R} - \{\pm 2\}$

Zéros : $x=0 \Leftrightarrow x^3 = 0$

	-2		0		2	
$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$

$$f(1) = \frac{1}{3\cdot(-1)} < 0$$

$$f(0) = \frac{0}{(0+2)(0-2)}$$

② A.V. $$\lim_{x\to 2} f(x) = \text{«}\frac{2^3}{(2+2)\underbrace{(2-2)}_{0}}\text{»} = \text{«}\frac{8}{0}\text{»} = \infty$$

$$= \frac{0}{2\cdot(-2)} = \frac{0}{-4} = 0$$

$$\Rightarrow \boxed{A.V. \text{ en } x=2}$$

$$\lim_{x\to -2} f(x) = \text{«}\frac{(-2)^3}{(-2+2)(-2-2)}\text{»} = \text{«}\frac{-8}{0\cdot(-4)}\text{»} = \text{«}\frac{-8}{0}\text{»} = \infty$$

$$\Rightarrow \boxed{A.V. \text{ en } x=-2}$$

$$f(x) = \frac{x^3}{x^2-4}$$

$\deg(x^3) = 3$

$\deg(x^2-4) = 2$

1 ← différence (haut - bas)

$\Rightarrow$ A.O.

$$\begin{array}{l|l} x^3 & x^2-4 \\ \hline x^3-4x & x \\ \hline 4x & \end{array}$$

A.O. en $y = 1\cdot x + 0$

$$d(x) = \frac{4x}{x^2-4} = \frac{4x}{(x+2)(x-2)}$$

← différence entre f et l'A.O.

x		-2		0		2	
$d(x)$	$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$
	dessous						dessus

(3) $f(x) = \dfrac{x^3}{x^2-4}$ $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$$f'(x) = \frac{(x^3)' \cdot (x^2-4) - x^3 \cdot (x^2-4)'}{(x^2-4)^2}$$

$$= \frac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2}$$

$$= \frac{3x^4 - 12x^2 - 2x^4}{(x^2-4)^2} = \frac{x^4 - 12x^2}{(x^2-4)^2}$$

$$f'(1) = \frac{1^2(1^2-12)}{(1^2-4)^2} < 0$$

$$= \frac{x^2(x^2-12)}{(x^2-4)^2} = \frac{x^2(x-\sqrt{12})^1(x+\sqrt{12})^1}{(x-2)^2(x+2)^2}$$

$$\boxed{\sqrt{12} \simeq 3{,}46 \simeq 3{,}5}$$

Zeros de $f'(x)$: $x=0$ / $x \simeq 3{,}5$ / $x \simeq -3{,}5$

$$x^2(x-\sqrt{12})(x+\sqrt{12}) = 0$$

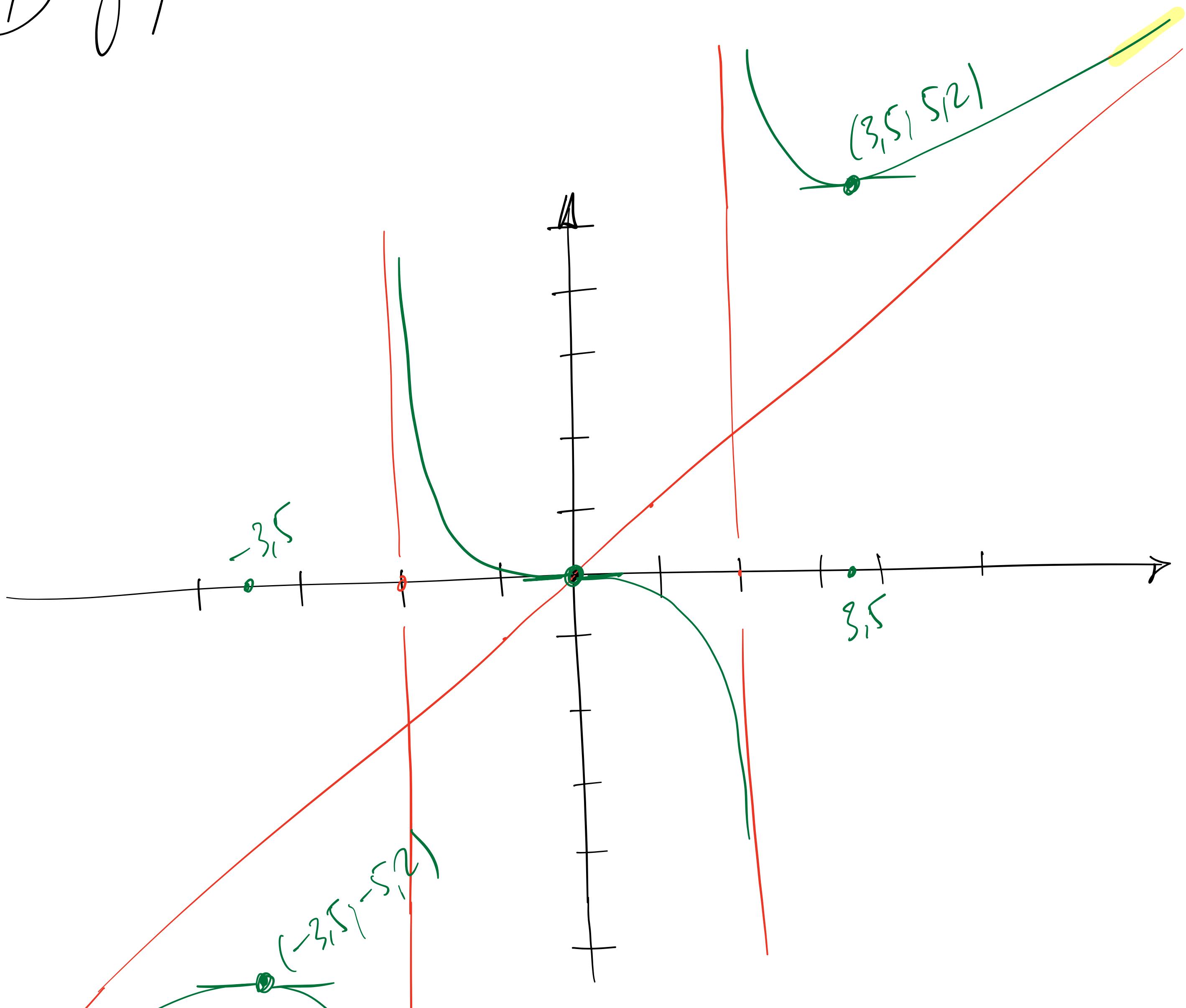
x		$-3{,}5$		-2		0		2		$3{,}5$	
f'	$+$	0	$-$	$\Vert$	$-$	0	$-$	$\Vert$	$-$	0	$+$
f	$\nearrow$	MAX	$\searrow$	$\Vert$	$\searrow$	PALIER	$\searrow$	$\Vert$	$\searrow$	MIN	$\nearrow$

$(-3{,}5;\ f(-3{,}5))$	$(0;\ 0)$	$(3{,}5;\ f(3{,}5))$
MAX	PALIER	MIN

$$f(-3{,}5) = \frac{(-3{,}5)^3}{(-3{,}5)^2-4} = -5{,}2$$

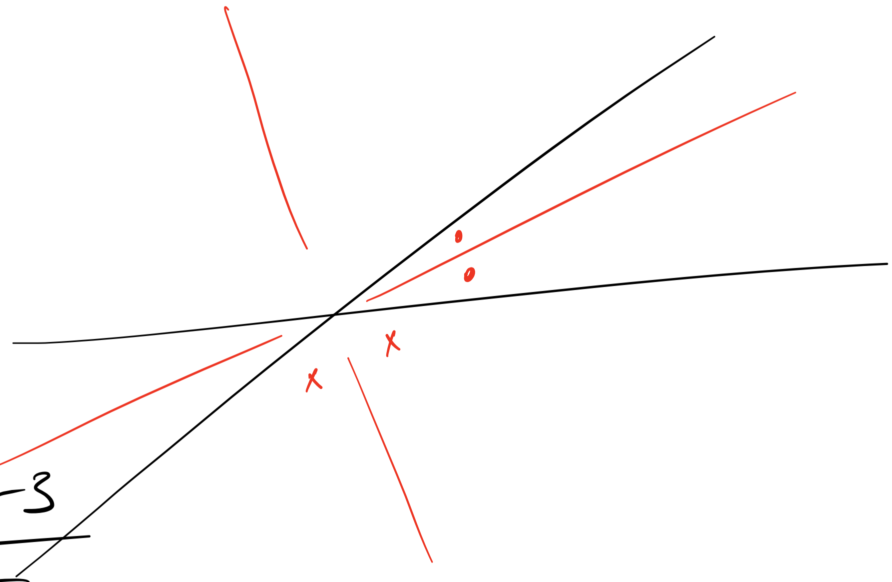
$$f(3{,}5) = \frac{3{,}5^3}{3{,}5^2-4} = 5{,}2$$

④ Graphe



$d_1: \; 3x + 4y - 2 = 0$

$d_2: \; 4x - 3y - 3 = 0$



$$\frac{3x+4y-2}{\sqrt{3^2+4^2}} = \pm \frac{4x-3y-3}{\sqrt{4^2+3^2}}$$

$$\Leftrightarrow \frac{3x+4y-2}{\sqrt{25}} = \pm \frac{4x-3y-3}{\sqrt{25}}$$

$$\Leftrightarrow \frac{3x+4y-2}{5} = \pm \frac{4x-3y-3}{5} \quad \Big\downarrow \cdot 5$$

$$\Leftrightarrow 3x+4y-2 = \pm(4x-3y-3)$$

$$3x+4y-2 = 4x-3y-3$$

$$-x+7y+1=0$$

$$3x+4y-2 = -(4x-3y-3)$$

$$3x+4y-2 = -4x+3y+3$$

$$\boxed{7x+y-5=0} \quad b_2$$

$$\boxed{x - 7y - 1 = 0} \quad b_1$$

$$2(x-1)(x+2) - (x-1)^2 \cdot 1 =$$

$$2\underbrace{(x-1)}(x+2) - \underbrace{(x-1)}(x-1) =$$

$$(x-1)\Big(2(x+2) - (x-1)\Big) =$$

$$(x-1)(2x+4 - x+1) = (x-1)(x+5)$$

$$2x\cdot\sqrt{27-x^2} + x^2\cdot\left(\sqrt{27-x^2}\right)' =$$

$$\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$$

$$2x\sqrt{27-x^2} + x^2\,\frac{(27-x^2)'}{2\sqrt{27-x^2}} =$$

$$2x\sqrt{27-x^2} + \boxed{x^2\cdot\frac{-\cancel{2}x}{\cancel{2}\sqrt{27-x^2}}} =$$

$$\frac{2x\sqrt{27-x^2}}{1}\cdot\frac{\sqrt{27-x^2}}{\sqrt{27-x^2}} - \boxed{\frac{x^3}{\sqrt{27-x^2}}} =$$

$$\frac{2x\left(\sqrt{27-x^2}\right)^2}{\sqrt{27-x^2}} - \frac{x^3}{\sqrt{27-x^2}} =$$

$$\frac{2x\,(27-x^2) - x^3}{\sqrt{27-x^2}} =$$

$$\frac{54x-2x^3-x^3}{\sqrt{27-x^2}} = \frac{54x-3x^3}{\sqrt{27-x^2}}$$

$$8x^3 - 18x = x \cdot (\underbrace{8x^2 - 18}_{\text{Viète}}) = 0$$

$$x = 0$$

$$(8x^2 - 18) = 0$$