

1.5.4

$$\cos(3\theta)$$

Montrer:

$$[1; \theta]^3 = [1; 3\theta]$$

$$[1; \theta]^3 = (\cos \theta + i \sin \theta)^3 =$$

$$= \cos^3 \theta + 3 \cos^2 \theta i \sin \theta + 3 \cos \theta (i \sin \theta)^2 + (i \sin \theta)^3$$

$$= \cos^3 \theta - 3 \cos \theta \sin^2 \theta + i (3 \cos^2 \theta \sin \theta - \sin^3 \theta)$$

$$= [1; 3\theta] = \cos 3\theta + i \sin 3\theta$$

Donc :

$$\begin{cases} \cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta = \cos^3 \theta - 3 \cos \theta (1 - \cos^2 \theta) \\ \sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta \end{cases}$$

$$\Rightarrow \cos 3\theta = \cos^3 \theta - 3 \cos \theta + 3 \cos^3 \theta = 4 \cos^3 \theta - 3 \cos \theta$$

Then: $[r; \theta]^n = [r^n; n\theta]$ si $n \geq 1$ $r > 0$

$$[r; \theta]^{-n} = \frac{1}{[r; \theta]^n} = \frac{1}{[r^n; n\theta]} = \frac{[1; 0]}{[r^n; n\theta]}$$

$$= \left[\frac{1}{r^n}; 0 - n\theta \right] = \left[\frac{1}{r^n}; -n\theta \right]$$


Moivre

Calcul de $\sqrt{i}$:

$$(x+iy) = \sqrt{i} \in \mathbb{C}$$
$$x, y \in \mathbb{R}$$

$$(x+iy)^2 = i$$

$$\Leftrightarrow x^2 + 2xyi + (iy)^2 = i$$

$$\Leftrightarrow x^2 + i^2 y^2 + 2xyi = i$$

$$\Leftrightarrow x^2 - y^2 + 2xyi = 0 + 1i$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 0 & (1) \\ 2xy = 1 \end{cases}$$

$$2xy = 1$$

x et y sont de même signe

$$\begin{cases} x^2 - y^2 = 0 & (1) \\ x^2 + y^2 = 1 & (2) \end{cases}$$

$(1)+(2)$

$$\Rightarrow 2x^2 - y^2 + y^2 = 1 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = \pm \frac{\sqrt{2}}{2}$$

En plus :

$$|(x+iy)|^2 = |i|$$
$$\rightarrow \sqrt{0^2 + 1^2}$$

$$|x+iy|^2 = 1$$

$$\left(\sqrt{x^2 + y^2}\right)^2 = 1$$

$$x^2 + y^2 = 1$$

$$\textcircled{2} - \textcircled{1} \Rightarrow x^2 + y^2 - (x^2 - y^2) = 1 - 0$$

$$x^2 + y^2 - x^2 + y^2 = 1 \quad \frac{1}{\sqrt{2}} = y = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$2y^2 = 1 \Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm \frac{\sqrt{2}}{2}$$

$\Rightarrow$ Les deux racines carrées de i sont :

$$\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \quad -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$$

$$\frac{1+i}{\sqrt{2}}$$

$$\frac{-1-i}{\sqrt{2}}$$

$$3 \quad 2 \quad 7 \quad -20$$

$$P(z) = 0$$

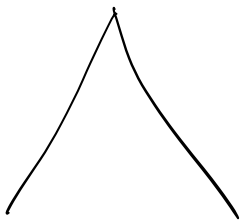
$$\begin{array}{r} -1+2i \quad -3+6i \quad -11-8i \\ \hline 3 \quad -1+6i \quad -4-8i \end{array}$$

$$\Leftrightarrow z-2 \mid P(z)$$

$$(-1+2i)(-1+6i) = 1-12-8i = -11-8i$$

$$(-1+2i)(-4-8i) = 4+16+\underbrace{8i-8i}_{0i}$$

$$z^2 \cdot Q(z) = 0$$



$$z^2 = 0$$

$$Q(z) = 0$$

$$\Leftrightarrow z = 0$$

A résoudre

↑
sol.

$$z^4 + z^2 + 1 = z^4 + 2z^2 + 1 - z^2$$

$$t^2 + t + 1$$

$$= (z^2 + 1)^2 - z^2$$

$$= (z^2 + 1 + z)(z^2 + 1 - z)$$

$$= (z^2 + z + 1)(z^2 - z + 1)$$

$$z = \frac{-1 \pm i\sqrt{3}}{2}$$

$$z = \frac{1 \pm i\sqrt{3}}{2}$$

$$= \left(z - \frac{-1 + i\sqrt{3}}{2}\right) \left(z - \frac{-1 - i\sqrt{3}}{2}\right)$$

$$\left(z - \frac{1 + i\sqrt{3}}{2}\right) \left(z - \frac{1 - i\sqrt{3}}{2}\right)$$