Def. Soit u_n une suite de $\mathbb{R}$.

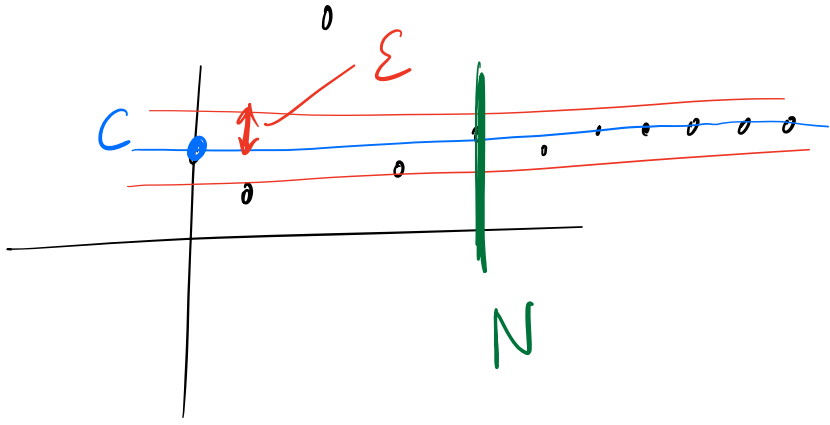
On dit que u_n converge vers $c \in \mathbb{R}$, noté

$$\lim u_n = c = \lim_{n \to +\infty} u_n$$

si $\forall \varepsilon > 0 \ \exists N$

tq. si $n \geq N$

$$|u_n - c| < \varepsilon$$



Prop. Soit $a, b \in \mathbb{R}$.

$$|a+b| \leq |a| + |b|$$

(inégalité triangulaire)

$$|x| = \begin{cases} x \text{ si } x \geq 0 \\ -x \text{ sinon} \end{cases}$$

Preuve: On observe que $a \cdot b \leq |a| \cdot |b|$

($a, b \geq 0$ ✓

$a, b \leq 0$ ✓

$a \geq 0 \ b \leq 0$ ✓

$a \leq 0 \ b \geq 0$ ✓)

$$(a+b)^2 = a^2 + 2ab + b^2 = |a|^2 + 2ab + |b|^2$$

$$\leq |a|^2 + 2|a||b| + |b|^2 = (|a| + |b|)^2$$

$$\sqrt{(a+b)^2} \leq \sqrt{(|a| + |b|)^2}$$

$$\sqrt{x^2} = |x|$$

$$|a+b| \leq |a| + |b|$$

CQFD

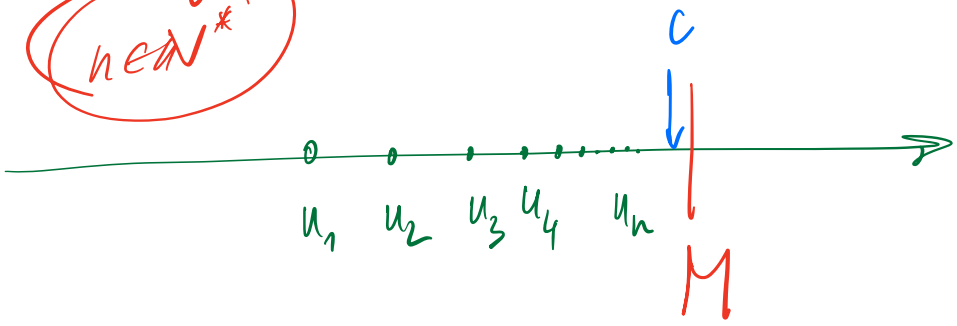
Def. Une suite u_n de $\mathbb{R}$ est

majorée si $u_n \leq M$ $\forall n$ pour $M \in \mathbb{R}$

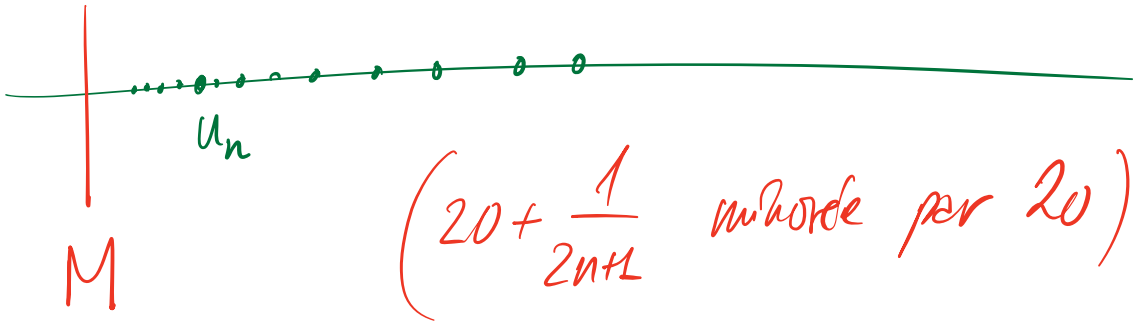
$\left(\frac{1}{n}\right.$ maj. par 1$\left.\right)$

$$\sum_{n=0}^{k} \frac{1}{2^n} = u_k$$

$n \in \mathbb{N}^*$



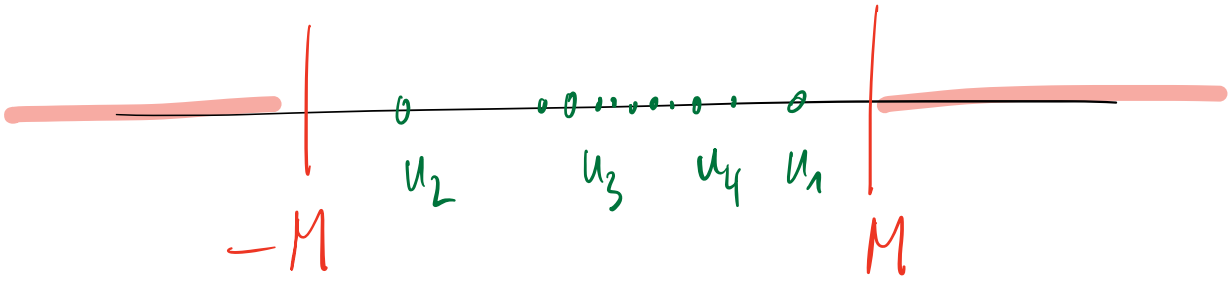
minorée si $u_n \geq M$ $\forall n$ pour $M \in \mathbb{R}$



$\left(20 + \frac{1}{2n+1}\right.$ minorée par 20$\left.\right)$

bornée si $|u_n| \leq M$ pour $M \in]0;+\infty[$

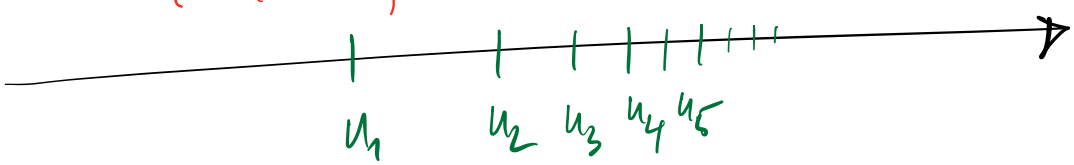
$\Leftrightarrow -M \leq u_n \leq M$



croissante si $u_n \leq u_{n+1}$ $\forall n$

strictement croissante si $u_n < u_{n+1}$

($u_n = n^2$, $u_n = n+1$, $u_n = 2^n$)



décroissante si $u_n \geq u_{n+1}$ $\forall n$

strict. décroissante si $u_n > u_{n+1}$

($u_n = \frac{1}{n}$, $u_n = \frac{1}{n^k}$, $u_n = 1-n$)

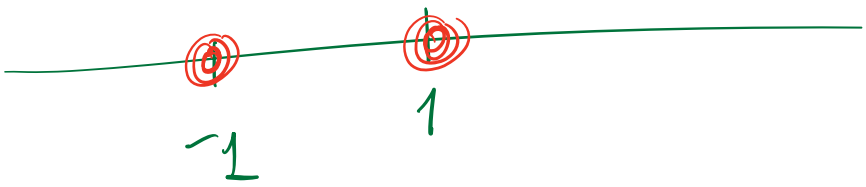
Prop. Soit u_n une suite de $\mathbb{R}$ tq. $\lim u_n = c$

Alors u_n est **bornée**.

La réciproque est fausse :

$u_n = (-1)^n$ est bornée

$|u_n| \leq |(-1)^n| = 1$



u_n diverge.

preuve : $u_n \xrightarrow[n\to\infty]{} c$ signifie que

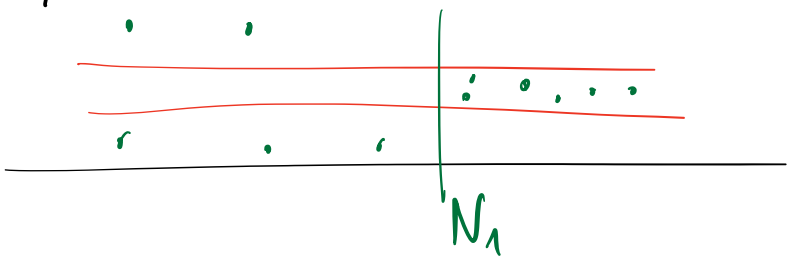
$\forall \varepsilon > 0 \ \exists N$ tq. $n \geq N \Rightarrow |u_n - c| < \varepsilon$

Posons $\varepsilon = 1$ $\exists N_1 \in \mathbb{N}$ tq. si $n \geq N_1$

alors $|u_n - c| < 1$ inégalité $\triangle$

$\Rightarrow |u_n| = |(u_n - c) + c| \leq |u_n - c| + |c| < 1 + |c|$

si $n \geq N_1$

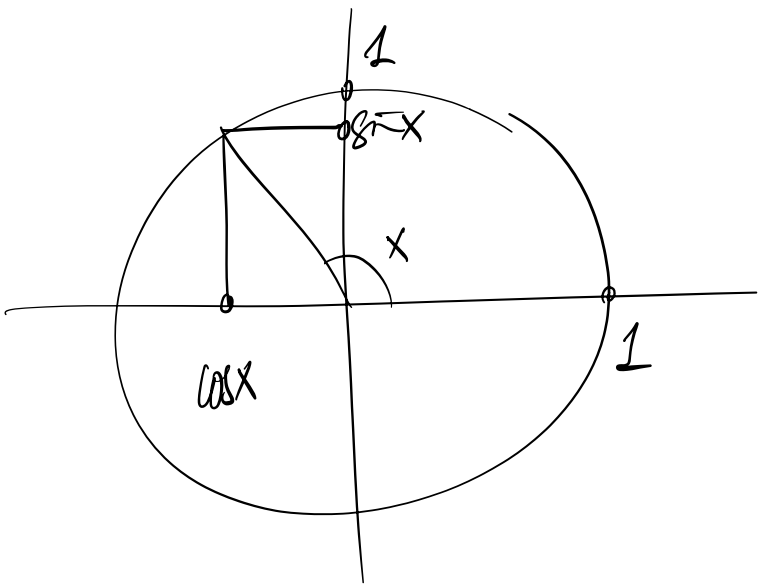


Soit $M = \max(|u_1|, |u_2|, \dots, |u_{N_1}|, 1+|c|) \geq 0$

$\Rightarrow |u_n| \leq M \quad \forall n \in \mathbb{N}$

CQFD

2.2.7



$-1 \leq \sin x \leq 1$

$-1 \leq \cos x \leq 1$

$$\frac{-1}{n^2} \leq \frac{\cos(n)}{n^2} \leq \frac{1}{n^2}$$

$n \to \infty$: 0 — $n \to \infty$: 0 — $n \to \infty$: 0

Thm des 2 gendarmes

$\frac{1}{n} \to 0$ ✓ $\Rightarrow \frac{1}{n^k} \to 0$ ✓

$\Rightarrow \frac{1}{n^3} \to 0$ ✓ $\Rightarrow 5 \cdot \frac{1}{n^3} \to 5 \cdot 0 \to 0$

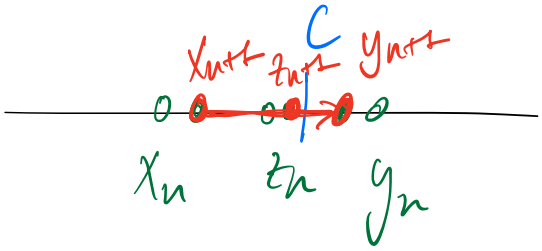
Théorème : (2 gendarmes)

Soit $x_n, y_n \xrightarrow{n\to\infty} c$

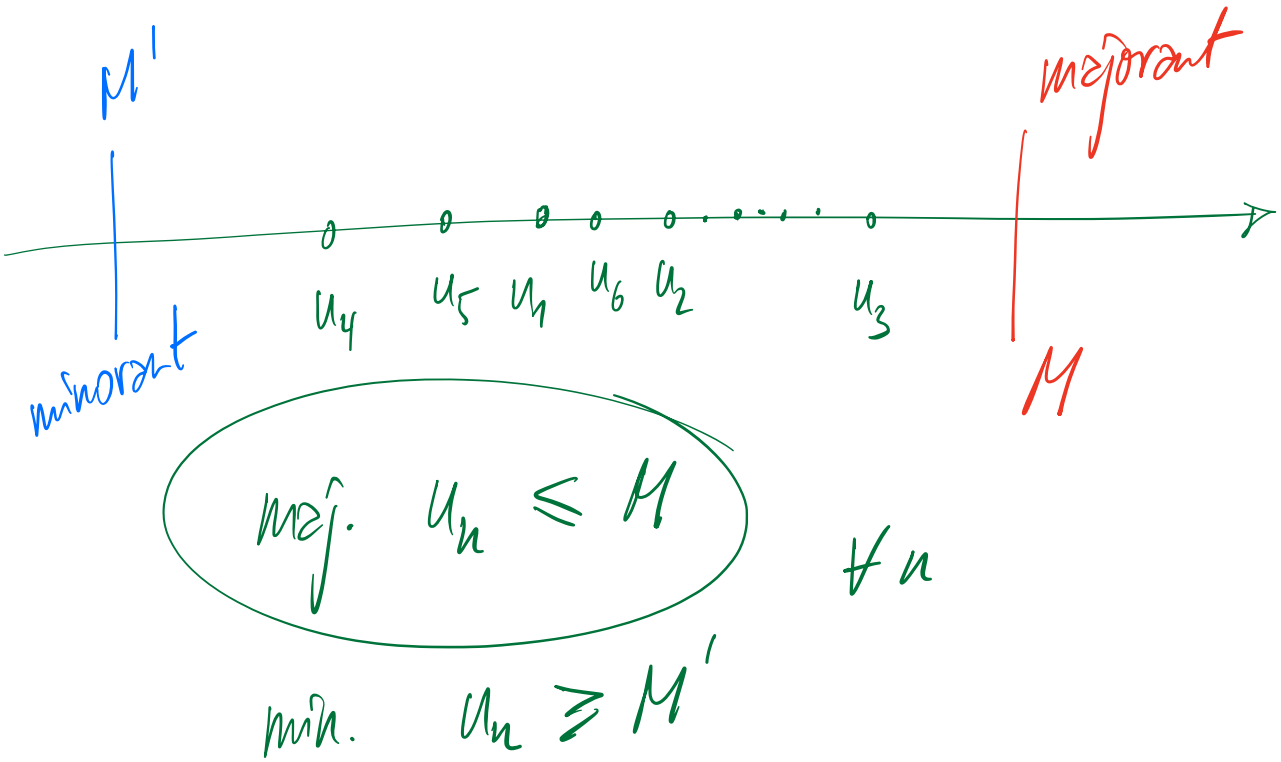
Si $x_n \le z_n \le y_n$ $\forall n \ge N$

c

alors $z_n \xrightarrow{n\to\infty} c$



2.2.6 à 2.2.8



maj. $u_n \leq M$ $\forall n$

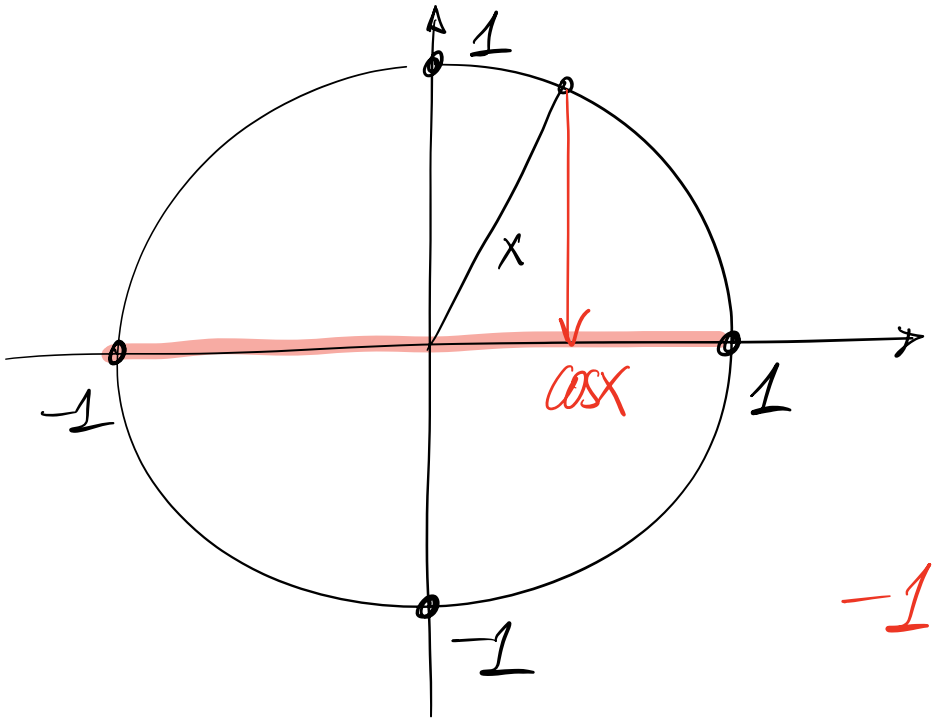
min. $u_n \geq M'$

$$\frac{(2n-1)^4}{(1-n^2)^2} = \frac{(2n)^4 + \dots}{n^4 + \dots}$$

$$= \frac{16 + \cdot \frac{}{}}{1 + \frac{}{}}$$

$$\downarrow n \to \infty$$

$$16$$



$$-1 \le \cos x \le 1$$

$$\forall x \in \mathbb{R}$$

$$\frac{-1}{n^2} \le \frac{\cos(n)}{n^2} \le \frac{1}{n^2}$$

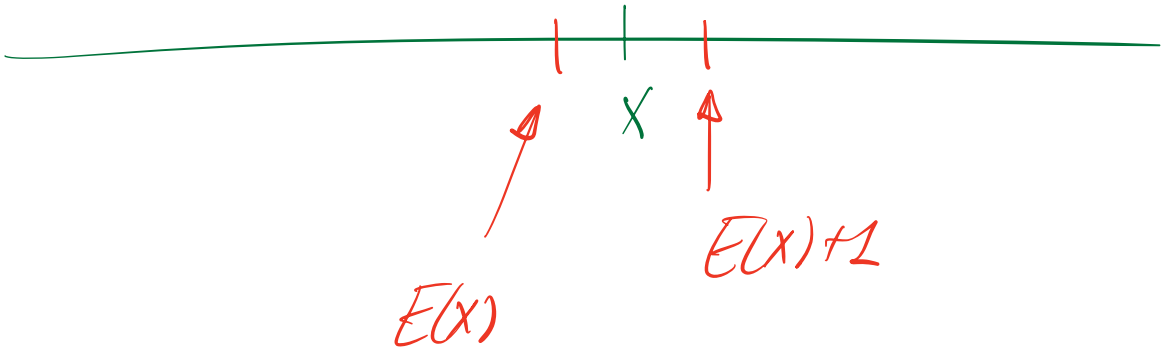
$$0 \le \frac{n!}{n^n} = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot \boxed{1}}{n \cdot n \cdot n \cdot n \quad \boxed{n}} \le \frac{1}{n}$$

$$1 \le 1 \quad \le 1 \quad \le 1$$

$$2 \cdot b \le 6$$

$$2 \le 1$$

$x \in \mathbb{R}$



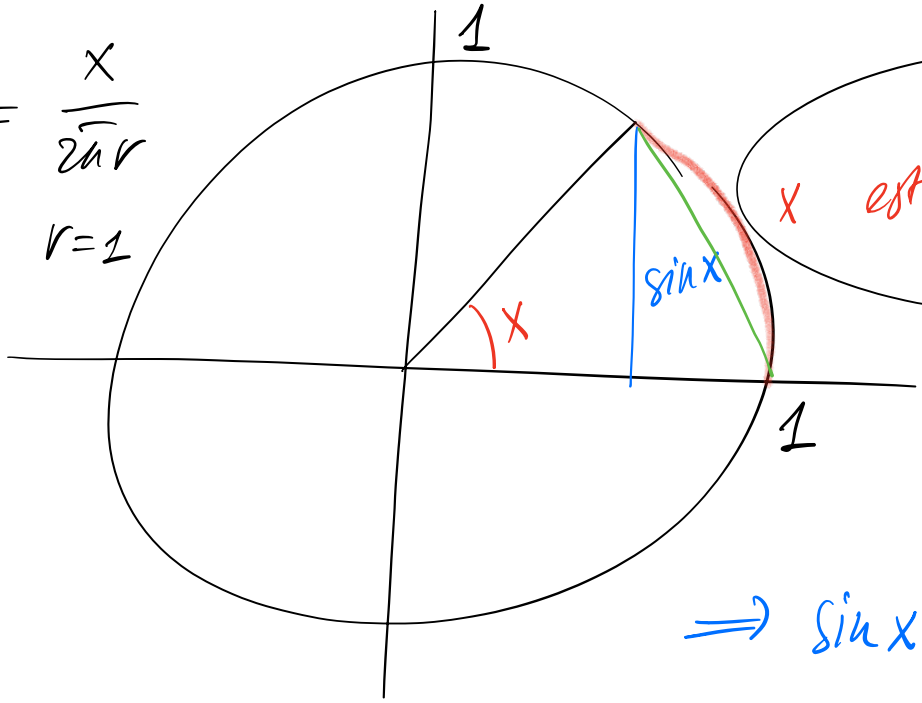
$E(x)$ est le plus grand nombre entier inférieur ou égal à x.

$E(5,9) = 5$ $\quad E(e) = 2$

$E(\pi) = 3$

$$\frac{X}{2\pi} = \frac{x}{2\pi r}$$

$r = 1$



x est la longueur d'arc

$$\Rightarrow \sin x \leq x$$

$$\Rightarrow 0 \leq n \cdot \sin\left(\frac{1}{n^2}\right) \leq n \cdot \frac{1}{n^2} = \frac{1}{n}$$

$\downarrow$ 0 | $\uparrow$ petit | $\downarrow$ 0

Théorème : Soit x_n et y_n deux suites de $\mathbb{R}$ tq.

$$\lim x_n = c \quad \text{et} \quad \lim y_n = d$$