

1.5.5

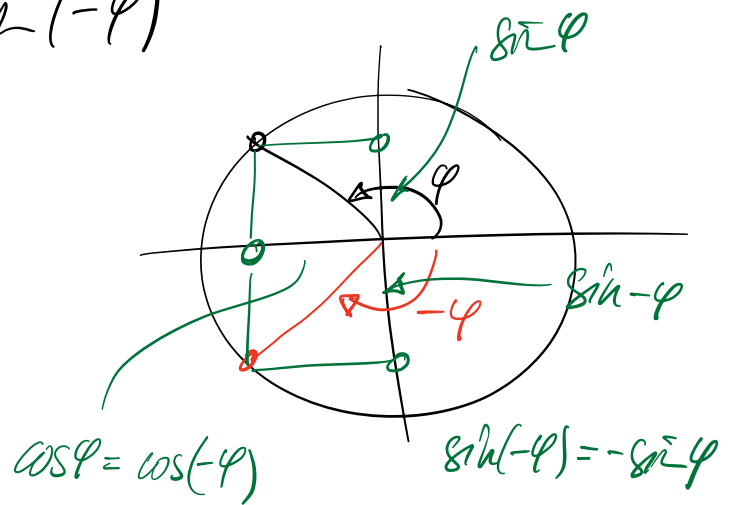
$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

Formule d'Euler

$$e^{i(-\varphi)} = \cos(-\varphi) + i \sin(-\varphi)$$

$$= \cos \varphi - i \sin \varphi$$

$$= \overline{e^{i\varphi}}$$



$$e^{i\varphi} + e^{i(-\varphi)} = \cos \varphi + i \sin \varphi + \cos \varphi - i \sin \varphi$$

$$= 2 \cos \varphi$$

 $z_1, z_2 \in \mathbb{C}$

$$e^{z_1} \cdot e^{z_2} = e^{z_1 + z_2}$$

Then

$$\cos^3 \varphi = \frac{(e^{i\varphi} + e^{-i\varphi})^3}{2^3}$$

$$e^z = \sum \frac{z^k}{k!}$$

$$= \frac{(e^{i\varphi})^3 + 3(e^{i\varphi})^2 \cdot (e^{-i\varphi}) + \dots}{8}$$

$$e \approx 2.718281\dots$$

$$e^z \stackrel{\text{def}}{=} \sum_{k=0}^{\infty} \frac{z^k}{k!} \quad \text{converge } \forall z \in \mathbb{C}$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

converge en 1700

$$e^{ix} = \sum_{k=0}^{\infty} \frac{(ix)^k}{k!}$$

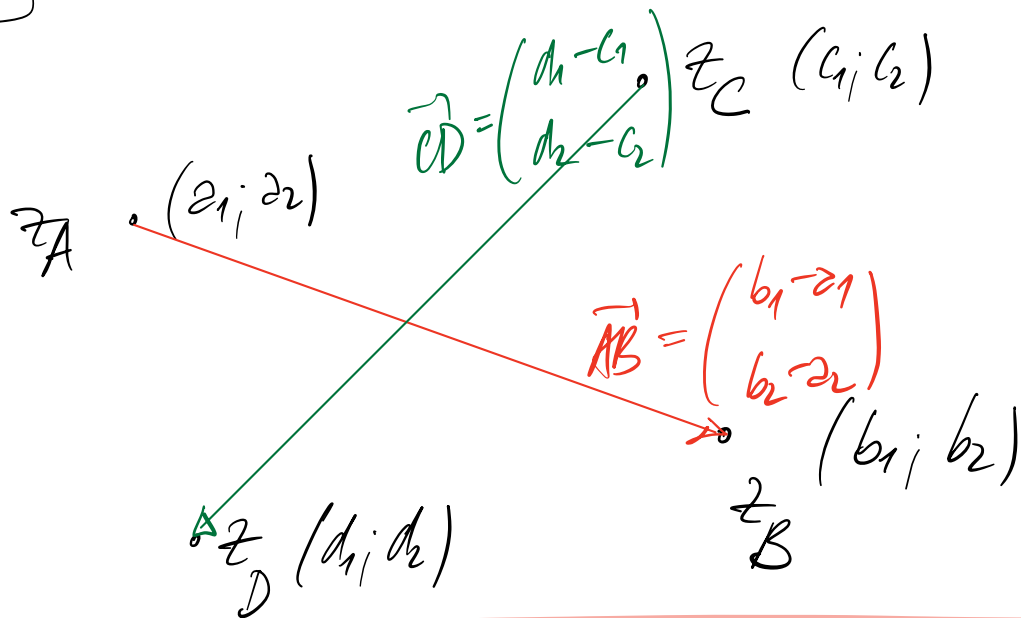
$$\begin{array}{cc} 1 & i^0 \\ i & i^1 \\ -1 & i^2 \\ -i & i^3 \\ & i^4 \end{array}$$

$$= 1 + \frac{i^1 x}{1!} + \frac{i^2 x^2}{2!} + \frac{i^3 x^3}{3!} + \frac{i^4 x^4}{4!} + \frac{i^5 x^5}{5!} + \dots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + i \cdot \left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right)$$

$$= \cos x + i \sin x$$

1.5.6

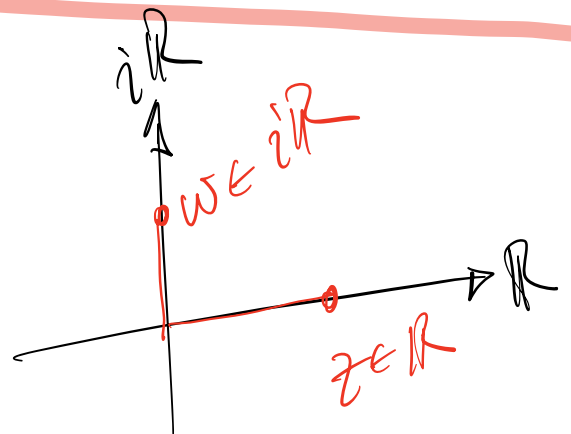


$$\vec{AB} \perp \vec{CD} \Leftrightarrow \vec{AB} \cdot \vec{CD} = 0$$

$$\Leftrightarrow (b_1 - z_1) \cdot (d_1 - c_1) + (b_2 - z_2) \cdot (d_2 - c_2) = 0$$

Compara:

$$E = \frac{z_D - z_C}{z_B - z_A} \in i \cdot \mathbb{R}^*$$



$$E = \frac{(d_1 + i d_2) - (c_1 + i c_2)}{(b_1 - i b_2) - (z_1 + i z_2)} = \frac{(d_1 - c_1) + i (d_2 - c_2)}{(b_1 - z_1) + i (b_2 - z_2)} \cdot \frac{(b_1 - z_1) - i (b_2 - z_2)}{(b_1 - z_1) - i (b_2 - z_2)}$$

$$(d_1 - c_1) \cdot (b_1 - a_1) + (d_2 - c_2)(b_2 - a_2) + i(d_2 - c_2)(b_1 - a_1) - (d_1 - c_1)(b_2 - a_2)$$

$$E = \frac{(b_1 - a_1)^2 + (b_2 - a_2)^2}{(b_1 - a_1)^2 + (b_2 - a_2)^2}$$

$$E \in i \cdot \mathbb{R}^* \Leftrightarrow E = x + iy \in i \cdot \mathbb{R}^*$$

$$\mathbb{R}^* = \mathbb{R} - \{0\}$$

$$\Leftrightarrow x=0 \text{ et } y \neq 0$$

$$\Leftrightarrow \frac{(d_1 - c_1)(b_1 - a_1) + (d_2 - c_2)(b_2 - a_2)}{(b_1 - a_1)^2 + (b_2 - a_2)^2} = 0$$

$$A=0 \Leftrightarrow B \neq 0$$

$$\text{et } \frac{(d_2 - c_2)(b_1 - a_1) - (d_1 - c_1)(b_2 - a_2)}{(b_1 - a_1)^2 + (b_2 - a_2)^2} \neq 0$$

$$\Leftrightarrow (b_1 - a_1)(d_1 - c_1) + (b_2 - a_2)(d_2 - c_2) = 0 \Leftrightarrow \overrightarrow{AB} \perp \overrightarrow{CD}$$

$$\text{et } \det(\overrightarrow{AB}; \overrightarrow{CD}) \neq 0 \neq \begin{vmatrix} b_1 - a_1 & d_1 - c_1 \\ b_2 - a_2 & d_2 - c_2 \end{vmatrix}$$

1.5.5

$$\cos^3 \varphi = \left(\frac{e^{i\varphi} + e^{-i\varphi}}{2} \right)^3 \in \mathbb{R}$$

$$\begin{aligned} \Leftrightarrow 8\cos^3 \varphi &= e^{i3\varphi} + 3e^{i2\varphi}e^{-i\varphi} + 3e^{i\varphi}e^{-i2\varphi} + e^{-i3\varphi} \\ &= \cos 3\varphi + i\sin 3\varphi + 3e^{i\varphi} + 3e^{-i\varphi} + e^{-i3\varphi} \\ &= \cancel{\cos 3\varphi} + \cancel{i\sin 3\varphi} + \cancel{3\cos \varphi} + \cancel{i3\sin \varphi} + \cancel{3\cos \varphi} - \cancel{i3\sin \varphi} \\ &\quad + \cancel{\cos 3\varphi} - \cancel{i\sin 3\varphi} \\ &= 2\cos 3\varphi + 6\cos \varphi \end{aligned}$$

$$\Leftrightarrow \boxed{\cos^3 \varphi = \frac{\cos 3\varphi + 3\cos \varphi}{4}}$$