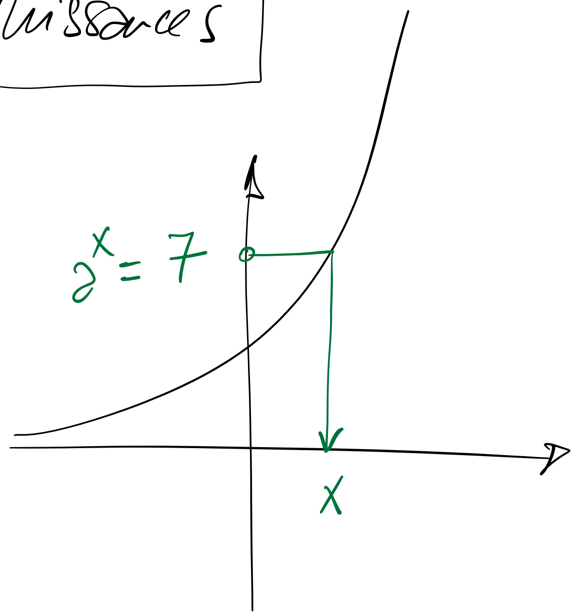
Puissances



a^x $\qquad a>0, \quad x \in \mathbb{R}$

Exemple: $f(x) = 2^x$

$2^5 \qquad 32$

$2^4 \qquad 16$

$2^\pi \longleftrightarrow 8{,}82$

$2^3 \qquad 8$

$2^x \qquad 7$

$2^2 \qquad 4$

$2^{\frac{3}{2}} \qquad \sqrt[2]{2^3}$

$2^1 \qquad 2$

$2^{\frac{1}{2}} \qquad \sqrt{2} \simeq 1{,}4142 \ldots$

$2^0 \qquad 1$

Fraction qcq.

$2^{\frac{p}{q}}$

2^{-1} $\quad \frac{1}{2}$

2^{-2} $\quad \frac{1}{4}$

$\boxed{\text{Logarithmes}}$

L'inconnue est « dans la puissance ».

$$2^{x} = 7 \iff x = \log_2(7) = \frac{\ln(7)}{\ln(2)} = \frac{\log(7)}{\log(2)}$$

base

Quelle est la puissance à laquelle élever 2 pour trouver 7 ?

$$3^{x} = 7 \iff \log_3(7)$$

$$\ln x = \log_e(x)$$

$e \simeq 2{,}718281\ldots$

$$\log(x) = \log_{10}(x)$$

TI 30

Prop.

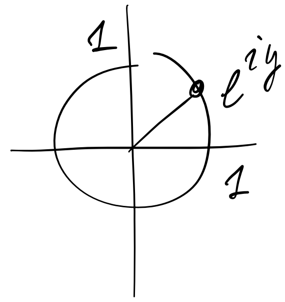
$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

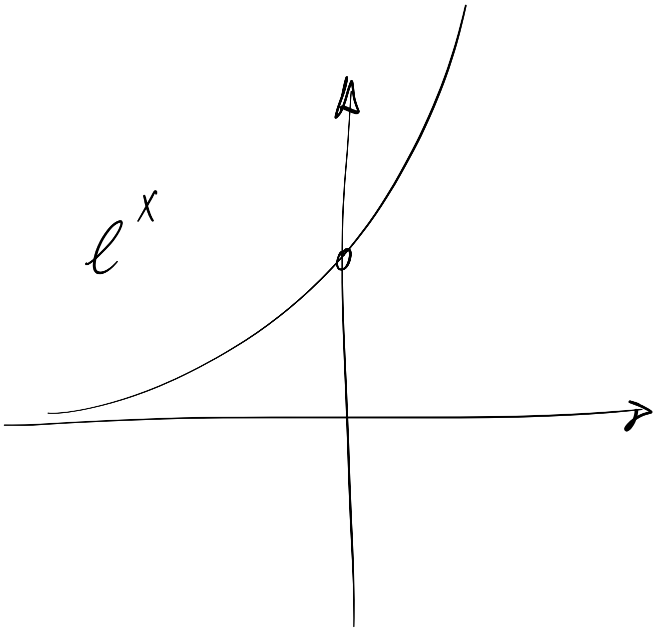
$$a^x \cdot a^y = a^{(x+y)}$$

$$\left(a^x\right)^y = a^{(xy)}$$

$$\boxed{a^x = u \iff x = \log_a(u)}$$

$$e^{i\pi} = \cos\pi + i\sin\pi$$





$$e \approx 2{,}718281\ldots$$

$$e^{i\cdot y} = \cos y + i\sin y$$

$$e^{x+iy} = e^x\cdot(\cos y + i\sin y)$$

$$e^{x+i\cdot 0} = e^x\cdot(\underbrace{\cos 0}_{1} + i\underbrace{\sin 0}_{0}) = e^x$$

Propriétés : $\boxed{2^x \cdot 2^y = 2^{(x+y)}}$

$$(2^x)^y = 2^{xy}$$

$$\boxed{\begin{array}{l} 2^{\log_2(x)} = x \\ \log_2(2^x) = x \end{array}}$$

Prop. $\log_2(x \cdot y) = \log_2(x) + \log_2(y)$

Preuve: $\log_2\left(2^{\log_2(x)} \cdot 2^{\log_2(y)}\right) = \log_2\left(2^{(\log_2(x) + \log_2(y))}\right)$

x $\qquad$ y

$= \log_2(x) + \log_2(y)$

CQFD

$$\boxed{2^x = u \iff x = \log_2(u)}$$

$$\Rightarrow 2^{\log_2(u)} = u \Rightarrow x = \log_2(2^x)$$

Prop. $\log_2(x^r) = r \cdot \log_2(x)$

Preuve:

$$\log_2(x^r) = \log_2\left(\left(2^{\log_2(x)}\right)^r\right)$$

$x = 2^{\log_2(x)}$

$$= \log_2\left(2^{\overbrace{(r\cdot\log_2(x))}^{t}}\right)$$

$(2^x)^y = 2^{xy}$

$\log_2(2^t) = t$

cas général

$$= r\cdot\log_2(x)$$

Exemples:

$\log_2 2 = 1$

$$\log_3(4^5) = 5\log_3 4$$

$$\log_3(81) = \log_3(3^4) = 4\log_3 3 = 4$$