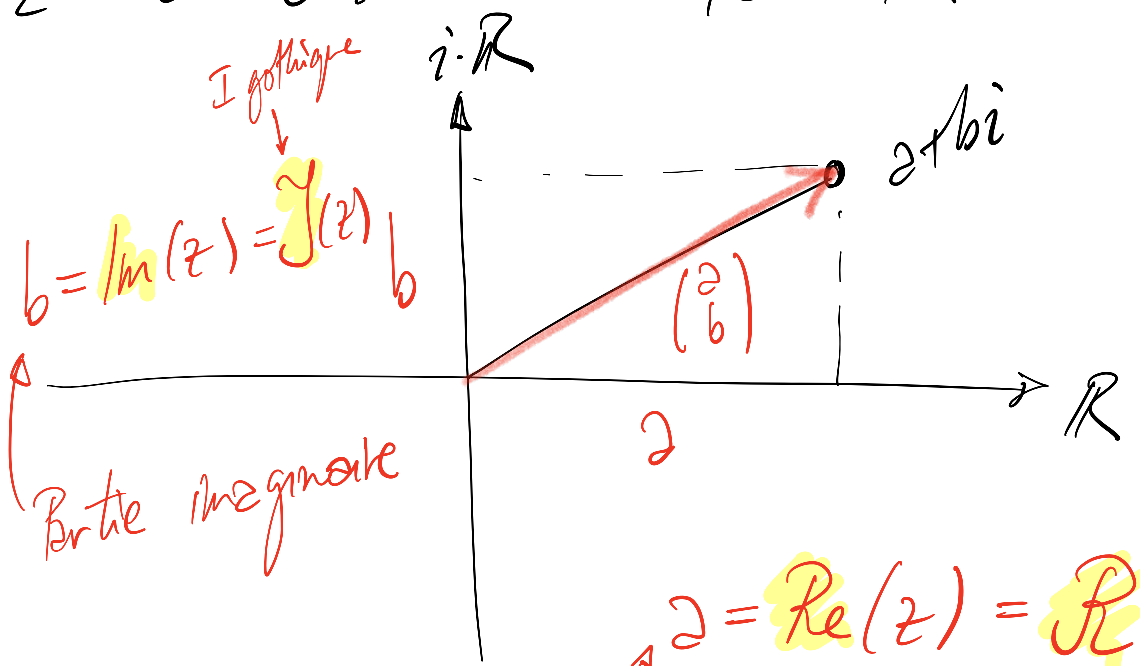
# $\mathbb{C}$ Nombres Complexes

$$z = a + b \cdot i \qquad a, b \in \mathbb{R}$$




$a = \mathrm{Re}(z) = \mathfrak{R}(z)$

Partie réelle

$$\overline{z} = a - bi$$

$$z + \overline{z} = \mathrm{Re}(z) \cdot 2$$
$$z - \overline{z} = \mathrm{Im}(z) \cdot 2i$$

À vérifier

$$|z| = \sqrt{a^2 + b^2}$$

Module de $z$

Si $z = x + 0i \in \mathbb{R}$

$$|z| = \sqrt{x^2} = |x|$$

Module

Valeur absolue

$$|x| = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{sinon} \end{cases}$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$|(a_1 + b_1 i)(a_2 + b_2 i)| = |a_1 + b_1 i| \cdot |a_2 + b_2 i|$$

$$\sqrt{(a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + a_2 b_1)^2} \stackrel{?}{=} \sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}$$

$$\Leftrightarrow (a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + a_2 b_1)^2 \stackrel{?}{=} (a_1^2 + b_1^2)(a_2^2 + b_2^2)$$

$$a_1^2 a_2^2 + b_1^2 b_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2 + \cancel{2 a_1 a_2 b_1 b_2} - \cancel{2 a_1 a_2 b_1 b_2} = a_1^2 a_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2 + b_1^2 b_2^2$$

✓

$$\sqrt[3]{5\sqrt{5\sqrt{5}}} =$$

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[n]{a^n} = a$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$$

$$\sqrt[3]{5\sqrt{\sqrt{5^2}\sqrt{5}}} =$$

$$\sqrt[3]{5\sqrt{\sqrt{5^3}}} = \sqrt[3]{5\sqrt[4]{5^3}}$$

$$= \sqrt[3]{\sqrt[4]{5^4}\,\sqrt[4]{5^3}}$$

$$= \sqrt[3]{\sqrt[4]{5^7}}$$

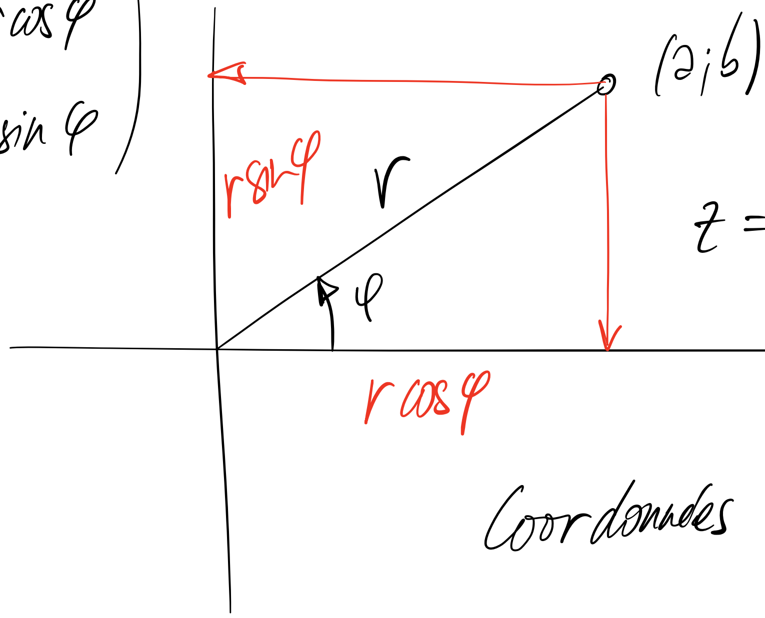
$$= \sqrt[12]{5^7}$$

$$\left(5^1\left(5^1 5^{1/2}\right)^{1/2}\right)^{1/3} = \left(5^1\cdot\left(5^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}$$

$$= \left(5^1\cdot 5^{\frac{3}{4}}\right)^{\frac{1}{3}}$$

$$= \left(5^{\frac{7}{4}}\right)^{\frac{1}{3}} = 5^{\frac{7}{12}} = \sqrt[12]{5^7}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \end{pmatrix}$$




$$r = \sqrt{a^2+b^2}$$

$$z = a+bi = r(\cos\varphi + i\sin\varphi)$$

Coordonnées cartésiennes : $(a;b)$

polaires : $(r;\varphi)$

$$z = [r;\varphi]$$

$\boxed{\text{Proposition}}$ Soit $z_1, z_2 \in \mathbb{C}$

$z_1 = a_1 + b_1 i$

$z_2 = a_2 + b_2 i$

$$|z_1| \cdot |z_2| = |z_1 \cdot z_2|$$

$$|a+bi| = \underbrace{\sqrt{a^2+b^2}}_{\in \mathbb{R}_+}$$

preuve:

$$|z_1| = \sqrt{a_1^2 + b_1^2}$$

$$|z_2| = \sqrt{a_2^2 + b_2^2}$$

$$z_1 \cdot z_2 = (a_1 + b_1 i)(a_2 + b_2 i)$$

$$= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i$$

$$|z_1 z_2| = \sqrt{(a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + a_2 b_1)^2}$$

$$= \sqrt{a_1^2 a_2^2 - \cancel{2 a_1 a_2 b_1 b_2} + b_1^2 b_2^2 + a_1^2 b_2^2 + \cancel{2 a_1 a_2 b_1 b_2} + a_2^2 b_1^2}$$

$$\boxed{= \sqrt{a_1^2 a_2^2 + b_1^2 b_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2}}$$

$$|z_1| \cdot |z_2| = \sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}$$

$$= \sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)} = \boxed{\sqrt{a_1^2 a_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2 + b_1^2 b_2^2}}$$

$$\Rightarrow |z_1| \cdot |z_2| = |z_1 \cdot z_2|$$

CQFD

Autre preuve

Vu que $|z_1|$, $|z_2|$, $|z_1 \cdot z_2| \in [0;+\infty[$,

il suffit de démontrer que

$$|z_1|^2 \cdot |z_2|^2 = |z_1 \cdot z_2|^2$$

$$\Leftrightarrow (a_1^2+b_1^2)(a_2^2+b_2^2) = \left[(a_1a_2-b_1b_2)^2+(a_1b_2+a_2b_1)^2\right]$$

$$\Leftrightarrow a_1^2a_2^2+a_1^2b_2^2+a_2^2b_1^2+b_1^2b_2^2 =$$

$$a_1^2a_2^2 \cancel{-2a_1a_2b_1b_2} + b_1^2b_2^2+a_1^2b_2^2 \cancel{+2a_1a_2b_1b_2} + a_2^2b_1^2$$

$$\Leftrightarrow a_1^2a_2^2+a_1^2b_2^2+a_2^2b_1^2+b_1^2b_2^2 = a_1^2a_2^2+a_1^2b_2^2+a_2^2b_1^2+b_1^2b_2^2$$

CQFD