

1.5.1

$$2) e^{-i\theta} \stackrel{\text{def}}{=} \cos(-\theta) + i\sin(-\theta)$$

$$\frac{1}{e^{i\theta}} = \frac{1}{\cos\theta + i\sin\theta} \cdot \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$$

$$= \frac{\cos\theta + i\sin(-\theta)}{\underbrace{\cos^2\theta + \sin^2\theta}_{=1}} \stackrel{\text{trigo 1M}}{=} \cos(-\theta) + i\sin(-\theta)$$

Moire sequis

$$e^{-i\theta} = [1; -\theta]$$

$$\frac{1}{e^{i\theta}} = \frac{1+0i}{[1; \theta]} = \frac{[1; 0]}{[1; \theta]} = \left[\frac{1}{1}; 0-\theta \right] = [1; -\theta]$$

1.3.1

$$e) \quad z^2 - 3(1+i) \cdot z + 6+7i = 0$$

$$1 \quad 2 \quad -3(1+i) \quad (6+7i)$$

$$z^2 + 6z + c = 0$$

trinôme du 2^{ème} degré

$$z = \frac{-(-3(1+i)) \pm \sqrt{(-3(1+i))^2 - 4 \cdot 1 \cdot (6+7i)}}{2}$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{3+3i \pm \sqrt{9(1+2i+i^2) - 24 - 28i}}{2}$$

$$a, b, c \in \mathbb{C}$$

$$z = \frac{3+3i \pm \sqrt{18i - 24 - 28i}}{2}$$

$$z = \frac{3+3i \pm \sqrt{-24 - 10i}}{2}$$

A' trouver

$$w^2 = -24 - 10i$$

$$a^2 - b^2 = -24$$

$$2ab = -10$$

$$a^2 + b^2 = \sqrt{24^2 + 10^2}$$

$$a^2 + b^2 = 26$$

$$2a^2 = 2 \quad a = \pm 1$$

$$a = 1 \Rightarrow b = -5$$

$$2b^2 = 50 \quad b = \pm 5$$

$$a = -1 \Rightarrow b = 5$$

$$\omega_1 = 1 - 5i$$

$$\omega_2 = -1 + 5i$$

$$\Rightarrow z = \frac{3 + 3i \pm (1 - 5i)}{2}$$

$$f_1 \quad \boxed{(1+2i)} z^2 \quad \boxed{-(7+4i)} z + \boxed{5-5i} = 0$$

$a \qquad b \qquad c$

$$az^2 + bz + c = 0 \quad \Rightarrow \quad z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = (7+4i)^2 - 4(1+2i)(5-5i)$$

$$= 33 + 56i - 20(3+i) = -27 + 36i = -9(3-4i)$$

$$= 9(-3+4i)$$

$$z = \frac{7+4i \pm \sqrt{9(-3+4i)}}{2(1+2i)} = \frac{7+4i \pm 3\sqrt{-3+4i}}{2(1+2i)}$$

$$w^2 = -3+4i$$

$$w = a+bi \rightarrow$$

$$\begin{cases} a^2+b^2 = 5 \\ a^2-b^2 = -3 \\ 2ab = 4 \end{cases}$$

$$$$

$$$$

Extraction

d'une
racine
complexe

$$\rightarrow (a+bi)^2 = (a^2-b^2) + 2abi = -3+4i$$

$$2a^2 = 2 \quad 2b^2 = 8$$

$$a = \pm 1 \quad b = \pm 2$$

$$w = \pm (1+2i)$$

$$\Rightarrow z = \frac{7+4i \pm 3(1+2i)}{2(1+2i)} = \frac{7+4i}{2(1+2i)} \pm \frac{3}{2}$$

$$= \frac{(7+4i)(1-2i)}{2(1+4)} \pm \frac{3}{2}$$

$$= \frac{5(3-2i)}{2 \cdot 5} \pm \frac{3}{2} = \frac{3-2i}{2} \pm \frac{3}{2}$$

$$= \begin{matrix} & 3 - i \\ & \swarrow \quad \searrow \\ & -i \end{matrix}$$

1.3.2

$$A^2 - B^2 = (A+B)(A-B)$$

$$\begin{aligned} a) \quad z^4 - 1 &= (z^2 - 1)(z^2 + 1) = (z^2 - 1)(z^2 - (-1)) \\ &= (z-1)(z+1)(z-i)(z+i) \end{aligned}$$

$$\text{car } \boxed{-1 = i^2 = (-i)^2}$$

$$\begin{aligned} b) \quad z^4 + 1 &= z^4 - (-1) \\ &= (z^2 - i)(z^2 + i) \end{aligned}$$