

TE 24/11/2026 } MR

TE 22/12/2026 }

TE 03/12/2026 OS

Suites de réels

$$(a_n)_{n \geq 0} \quad a_n \in \mathbb{R} \quad \forall n \in \mathbb{N}$$

Exemple:

$$\left(-15 \quad 11 \quad 10 \quad 1005 \quad 0 \quad 1 \right) \left[\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \dots \right]$$

Le fin de la suite compte

Def. Soit $c \in \mathbb{R}$. On dit que la limite de a_n est c ou que a_n converge vers c si

$$\forall \varepsilon > 0 \exists N_\varepsilon \in \mathbb{N} \text{ tq.}$$

$$n \geq N_\varepsilon \Rightarrow |a_n - c| < \varepsilon$$

Not. $\lim_{n \rightarrow \infty} a_n = c$

ε
0,1
0,01
0,001
0,0001
0,00001
0,000001
⋮

Exemple: $a_n = \frac{1}{n}$ $n \in \mathbb{N}^*$

À démontrer: $\lim a_n = \lim \frac{1}{n} = 0$

Si c'est vrai: soit $\varepsilon > 0$

$$\left| \frac{1}{n} - 0 \right| < \varepsilon \quad \text{si } n \text{ assez grand}$$

$$\Leftrightarrow \left| \frac{1}{n} \right| < \varepsilon$$

$n > 0$

$$\Leftrightarrow \frac{1}{n} < \varepsilon$$

$n > 0$

$$\Leftrightarrow 1 < \varepsilon n$$

$\varepsilon > 0$

$$\Leftrightarrow \frac{1}{\varepsilon} < n$$

$$\Rightarrow N_\varepsilon = E\left(\frac{1}{\varepsilon}\right) + 1$$

$$\left| \frac{1}{n} + 0i \right| = \sqrt{\left(\frac{1}{n}\right)^2 + 0^2} = \sqrt{\frac{1}{n^2}}$$

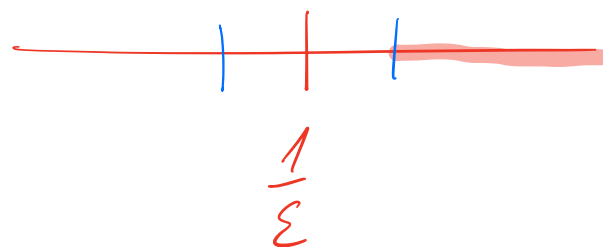
module

$$\left| \frac{1}{n} \right| = \begin{cases} \frac{1}{n} & \text{si } \frac{1}{n} \geq 0 \\ -\frac{1}{n} & \text{si } \frac{1}{n} < 0 \end{cases}$$

$$= \left| \frac{1}{n} \right| \quad \text{valeur absolue}$$

$$E\left(\frac{1}{\varepsilon}\right) \in \mathbb{N}$$

$$E\left(\frac{1}{\varepsilon}\right) + 1 \in \mathbb{N}$$



$$\boxed{\lim \frac{1}{n} = 0}$$

preuve: soit $\varepsilon > 0$ et $N_\varepsilon = E\left(\frac{1}{\varepsilon}\right) + 1$

Sint encore $n > N_\varepsilon$

$$\Rightarrow n > E\left(\frac{1}{\varepsilon}\right) + 1$$

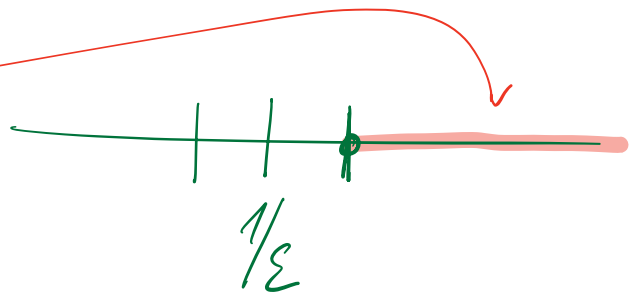
$$n > \frac{1}{\varepsilon}$$

$$\Rightarrow \frac{1}{n} < \varepsilon$$

$$\Rightarrow \left| \frac{1}{n} - 0 \right| < \varepsilon$$

$$\Rightarrow \forall \varepsilon > 0 \exists N_\varepsilon \text{ tq. } n \geq N_\varepsilon \Rightarrow \left| \frac{1}{n} - 0 \right| < \varepsilon$$

CQFD



2.2.5 $u_n, n \in \mathbb{N}$ et $n \geq 3$

Démontrer à l'aide de la définition

que $\lim u_n = 0$ si $u_n = \frac{3}{n-2}, n \geq 3$.

Rappel: Une suite u_n converge vers c si

$$\forall \varepsilon > 0 \exists N_\varepsilon \text{ tq. } n \geq N_\varepsilon \Rightarrow |u_n - c| < \varepsilon$$

preuve: Soit $\varepsilon > 0$

On suppose que c'est vrai:

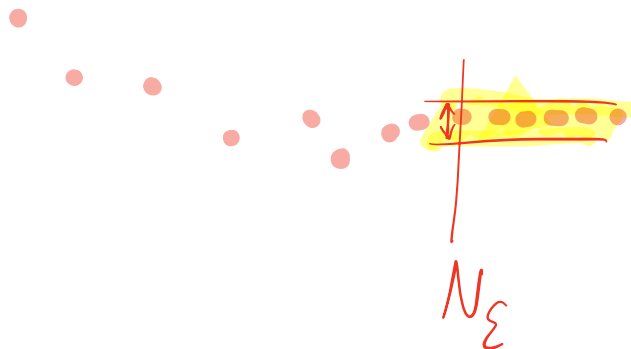
$$\left| \frac{3}{n-2} - 0 \right| < \varepsilon$$

$$\Leftrightarrow \left| \frac{3}{n-2} \right| < \varepsilon$$

$n-2 > 0$

$$\Leftrightarrow \frac{3}{n-2} < \varepsilon$$

$$n-2 > 0 \text{ si } n \geq 3$$



$$\varepsilon > 0, n-2 > 0$$



$$\frac{3}{\varepsilon} < n-2$$

$$\Leftrightarrow \frac{3}{\varepsilon} + 2 < n$$

$$N_{\varepsilon} = E\left(\frac{3}{\varepsilon} + 2\right) + 1$$

$$\Rightarrow N_{\varepsilon} = E\left(\frac{3}{\varepsilon} + 2\right) + 1$$

$$\text{si } n \geq N_{\varepsilon}$$

$$\Rightarrow n > \frac{3}{\varepsilon} + 2$$

$$\Rightarrow n-2 > \frac{3}{\varepsilon}$$

$$\Rightarrow \varepsilon > \frac{3}{n-2}$$

$$\Rightarrow \left| \frac{\varepsilon}{n-2} - 0 \right| < \varepsilon$$

CQFD

2.2.5

$$b) \lim \frac{n}{n+1} = 1$$

à démontrer à l'aide
de la déf.

$$\frac{1}{n} \longrightarrow 0$$

Thm

$$u_n = C \quad \forall n$$

$$\lim u_n = C$$

$\Rightarrow$

$$\frac{1}{n^k} \longrightarrow 0$$

$$k \geq 2$$

$$k \geq 1$$

$$k \in \mathbb{N}$$

$\frac{1}{n^k}$ est sm-s - suite de $\frac{1}{n}$

$$\frac{1}{n^2} : \quad 1 \quad \frac{1}{4} \quad \frac{1}{9} \quad \frac{1}{16} \quad \dots$$

Donc $\lim \frac{1}{n^3} = 0$

Thm : Si $u_n \rightarrow C$ et que $v_n \rightarrow d$,

alors (1) $\lim (u_n + v_n) = C + d = \lim u_n + \lim v_n$

(2) $\lim (u_n \cdot v_n) = C \cdot d = \lim u_n \cdot \lim v_n$

Si $d \neq 0$ (3) $\lim \left(\frac{u_n}{v_n} \right) = \frac{C}{d} = \frac{\lim u_n}{\lim v_n}$

Exemple:

$$u_n = \frac{1}{n} + 2 \xrightarrow{n \rightarrow \infty} 2$$

$$\begin{aligned} \lim\left(\frac{1}{n} + 2\right) &= \lim \frac{1}{n} + \lim 2 \\ &= 0 + 2 = 2 \end{aligned}$$

$$u_n = \frac{n^2 + 2}{n^3 + n^2 + n + 1}$$

$$u_n = \frac{\frac{1}{n^3} \cdot (n^2 + 2)}{\frac{1}{n^3} (n^3 + n^2 + n + 1)} = \frac{\frac{1}{n} + \frac{2}{n^3}}{1 + \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}} \xrightarrow{n \rightarrow \infty} \frac{0}{1} = 0$$

Handwritten notes:

- Red arrows from $\frac{1}{n}$ and $\frac{2}{n^3}$ in the numerator point to 0 and $2 \cdot 0$ respectively, with the note $n \rightarrow \infty$.
- Red arrows from $\frac{1}{n}$, $\frac{1}{n^2}$, and $\frac{1}{n^3}$ in the denominator point to 0 , 0 , and 0 respectively, with the note $n \rightarrow \infty$.
- Green arrow from "fraction rationnelle" points to the fraction $\frac{1}{n^3} \cdot (n^2 + 2)$.
- Green arrow from "nombre" points to the denominator $1 + \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}$.