

$$3 \cdot (-4) + 2(5) + 6 \cdot (3) = 16$$

$$8 \cdot (-4) + (5) - 2(3) = -33$$

$$5 \cdot (-4) + 5(5) + 3(3) = 14$$

$$3x + 2y + 6z = 16$$

$$8x + y - 2z = -33$$

$$5x + 5y + 3z = 14$$

$$\begin{pmatrix} 16 \\ -33 \\ 14 \end{pmatrix} = x \begin{pmatrix} 3 \\ 8 \\ 5 \end{pmatrix} + y \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + z \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$$

Diagram illustrating the vector equation with red arrows pointing from the vectors \vec{a} , \vec{b} , and \vec{c} to their respective columns in the matrix equation above.

$$\vec{v} = x \cdot \vec{a} + y \cdot \vec{b} + z \cdot \vec{c}$$

$$3x + 2y + 6z = 16$$

$$8x + y - 2z = -33$$

$$5x + 5y + 3z = 14$$

$\cdot (-3)$

$$3x + 2y + 6z = 16$$

$$-24x - 3y + 6z = 99$$

$$5x + 5y + 3z = 14$$

$$8L_1: 24x + 16y + 48z = 128$$

$$L_2 \leftarrow L_2 + 8L_1$$

$$3x + 2y + 6z = 16$$

$$13y + 54z = 227$$

$$5x + 5y + 3z = 14$$

$\cdot (-3)$

$$3x + 2y + 6z = 16$$

$$13y + 54z = 227$$

$$-15x - 15y - 9z = -42$$

$$L_3 \leftarrow L_3 + 5L_1$$

$$5L_1: 15x + 10y + 30z = 80$$

$$3x + 2y + 6z = 16$$

$$13y + 54z = 227$$

$$-5y + 21z = 38$$

$$\cdot 5$$

$$\cdot 13$$

$$13y = 227 - 162 \\ = 65$$

$$y = 5$$

$$270z + 273z = 5 \cdot 227 + 13 \cdot 38$$

$$543z = 1629$$

$$z = 3$$

$$3x + 2y + 6z = 16$$

$$y = 5 \quad / \quad z = 3$$

$$3x = 16 - 10 - 18 \\ = -12$$

$$S = \{(-4; 5; 3)\}$$

$$x = -4$$

$$\begin{pmatrix} 3 & 2 & 6 & 16 \\ 8 & 1 & -2 & -33 \\ 5 & 5 & 3 & 14 \end{pmatrix}$$

$$L_2 \leftarrow -3L_2 + 8L_1$$

$$\begin{pmatrix} 3 & 2 & 6 & 16 \\ 0 & 13 & 54 & 227 \\ 0 & -5 & 21 & 38 \end{pmatrix}$$

$$L_3 \leftarrow -3L_3 + 5L_1$$

$$\begin{pmatrix} 3 & 2 & 6 & 16 \\ 0 & 13 & 54 & 227 \\ 0 & 0 & 543 & 1629 \end{pmatrix}$$

$$L_3 \leftarrow 13L_3 + 5L_2$$