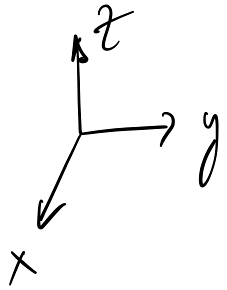
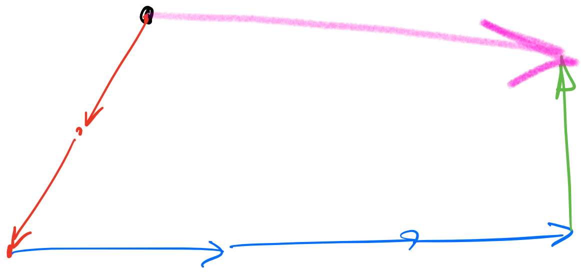
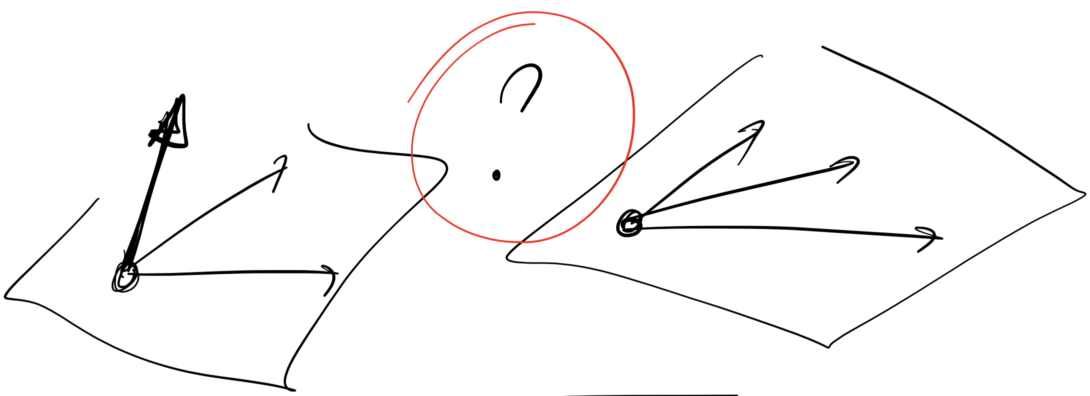
Coplanarité



$$\begin{array}{cl} \begin{pmatrix}2\\3\\1\end{pmatrix} & \begin{array}{l}\leftarrow x\\ \leftarrow y\\ \leftarrow z\end{array}\end{array}$$



$$\vec{u} = \begin{pmatrix}1\\2\\3\end{pmatrix} \qquad \vec{v} = \begin{pmatrix}2\\1\\-1\end{pmatrix} \qquad \vec{w} = \begin{pmatrix}3\\3\\2\end{pmatrix}$$



$\boxed{\vec{u},\ \vec{v} \text{ et } \vec{w} \text{ coplanaires ?}}$

Critère du déterminant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \end{pmatrix} = 1 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}$$

Changement de signe

« Au milieu » uniquement

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \end{pmatrix}$$

$$= 1 \cdot \left(1 \cdot 2 - (-1) \cdot 3\right) - 2\left(2 \cdot 2 - (-1) \cdot 3\right) + 3\left(2 \cdot 3 - 1 \cdot 3\right)$$

$$= 5 - 14 + 9 = 0$$

$\Rightarrow$ $\vec{u}$, $\vec{v}$ et $\vec{w}$ sont coplanaires, car

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \end{pmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

(si $\det(\vec{u}; \vec{v}, \vec{w}) \neq 0$, $\vec{u}$, $\vec{v}$ et $\vec{w}$ ne sont pas coplanaires)