P1 2018

a) Sujet : Éq. cartésienne d'un cercle

On cherche :

- le centre ;
- le rayon.

$$x^2+2x+y^2-6y=7$$

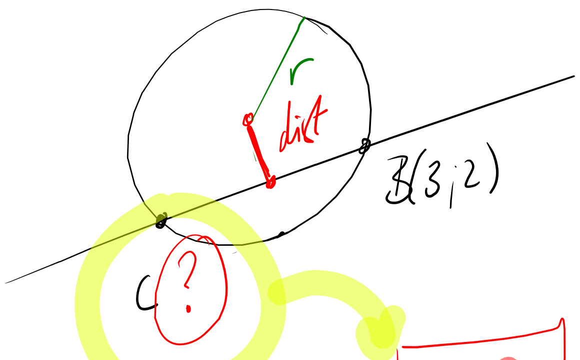
$$x^2+2\cdot x\cdot 1+y^2-2\cdot y\cdot 3=7$$

$$(x+1)^2+(y-3)^2-(1^2)-(3^2)=7$$

$$(x+1)^2+(y-3)^2=7+10=17$$

$$M(-1;3) \quad r=\sqrt{17}$$

b) Position relative de deux cercles, d'une droite et d'un cercle



$x-y-1=0 \Rightarrow \boxed{x=y+1}$

$(x+1)^2+(y-3)^2=17$

A résoudre

$\boxed{(y+2)^2+(y-3)^2=17}$

$\cancel{y^2}+\cancel{4y}+4+\cancel{y^2}-\cancel{6y}+9=17$

$2y^2-2y-4=0$

$y^2-y-2=0 \;/\; \underbrace{(y-2)}_{B(3;2)}(y+1)=0 \;/\; y=-1 \;/\; x=0$

$d: x-y-1=0$

B est sur d

$3-2-1=0 \Rightarrow \underline{B \in d}$

$\gamma: (x+1)^2+(y-3)^2=17 \qquad \Rightarrow B \in d \cap \gamma$

$(3+1)^2+(2-3)^2=17$

$16+1=17$ ✓ $\quad B \in \gamma$

$(0+1)^2+(-1-3)^2=17$ ✓

$C \in \gamma$

$\Rightarrow \boxed{C(0;-1)}$

$0-(-1)-1=0$ ✓

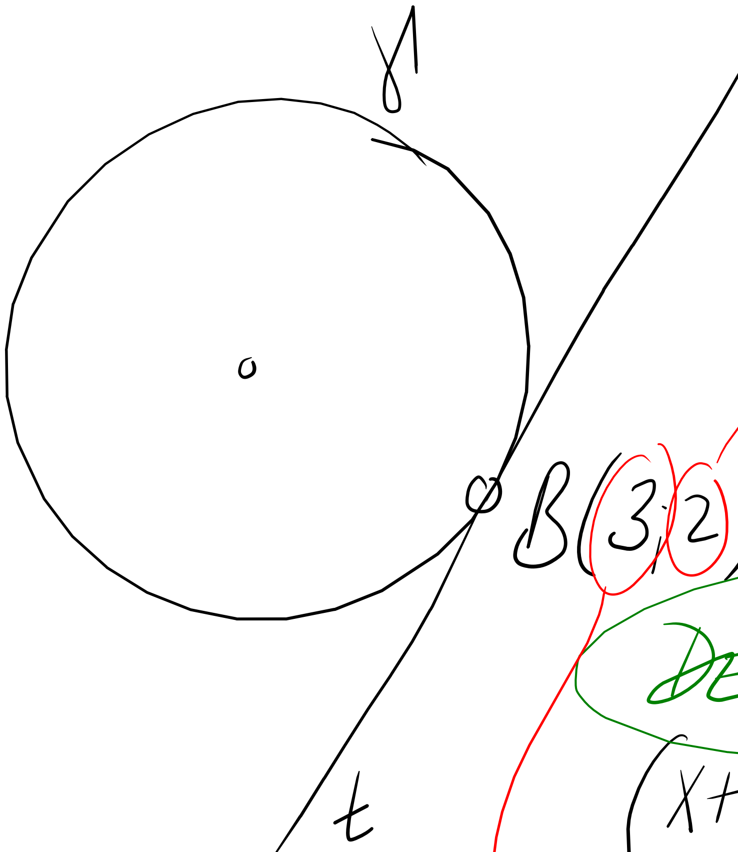
$C \in d$

On aurait aussi pu résoudre comme suit :

$$\begin{cases} x^2+y^2+2x-6y=7 \\ x-y-1=0 \end{cases}$$

c) t_B tangente à γ' au pt. B.

Tangente à un cercle par un pt. qcq.



$B(3;2) \in \gamma'$

DÉDOUBLER

$$(x+1)(x+1) + (y-3)(y-3) = 17$$

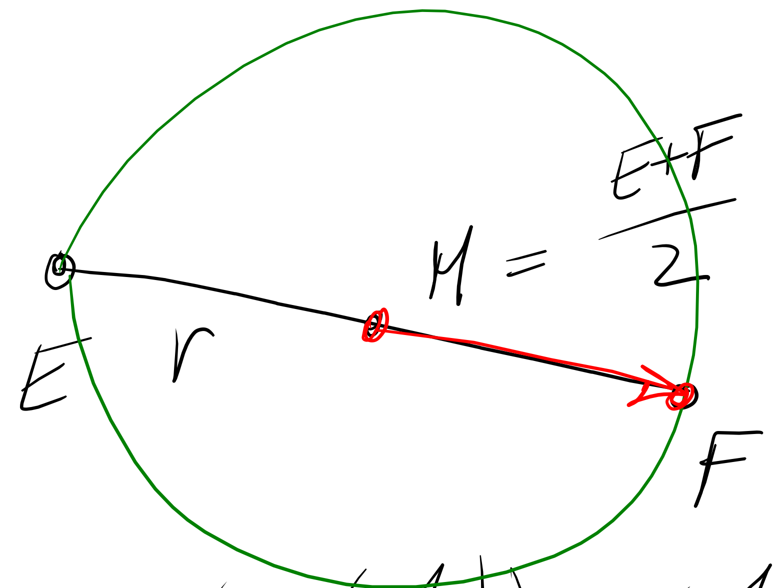
REMPLACER UNE LETTRE SUR DEUX

$$(x+1)\cdot\underbrace{4}_{3+1} + (y-3)\cdot\underbrace{(-1)}_{2-3} = 17$$

$$4x+4-y+3=17$$

$$t \quad \boxed{4x-y-10=0}$$

d) Éq. cart. d'un cercle



$M = \frac{E+F}{2} = \left(\frac{-1+0}{2}; \frac{\frac{7}{2}+(-\frac{1}{2})}{2}\right)$

$= \left(-\frac{1}{2}; \frac{3}{2}\right)$

Milieu

$\vec{MF} = \vec{OF} - \vec{OM}$

$\vec{MF} = \begin{pmatrix} 0-(-\frac{1}{2}) \\ -\frac{1}{2} - \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 1/2 \\ -2 \end{pmatrix}$

vecteur

norme

$\frac{1}{4} + \frac{4}{1} = \frac{1\cdot 1+4\cdot 4}{4\cdot 1}$

$r' = \|\vec{MF}\| = \sqrt{(1/2)^2 + (-2)^2} = \sqrt{1/4 + 4}$

$= \sqrt{\frac{17}{4}}$

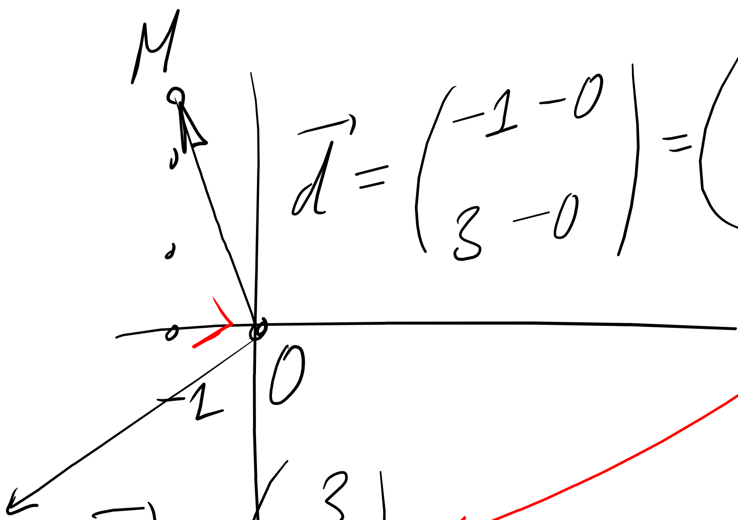
$\Rightarrow \gamma' : \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{3}{2}\right)^2 = \frac{17}{4}$

$M'\left(-\frac{1}{2}; \frac{3}{2}\right) \quad r' = \sqrt{\frac{17}{4}}$

e) Équation d'une droite

$$\boxed{d_{OM} : 3x+y+c=0}$$

par $(0;0)$



$$\vec{d} = \begin{pmatrix}-1-0\\ 3-0\end{pmatrix} = \begin{pmatrix}-1\\ 3\end{pmatrix}$$

$$\vec{n} = \begin{pmatrix}3\\ 1\end{pmatrix}$$

$$\vec{n} = \begin{pmatrix}a\\ b\end{pmatrix}$$

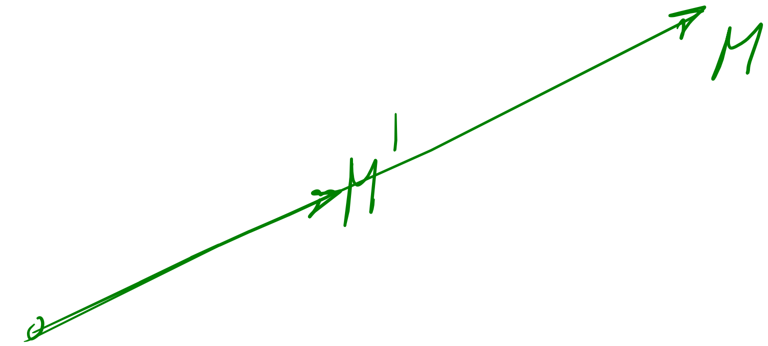
$$d : ax+by+c=0$$

$$3\cdot 0+0+c=0 \Rightarrow c=0 \Rightarrow d_{OM} : 3x+y=0$$

$O(0;0)$

$M(-1;3)$

$M'\left(-\frac{1}{2} ; \frac{3}{2}\right)$

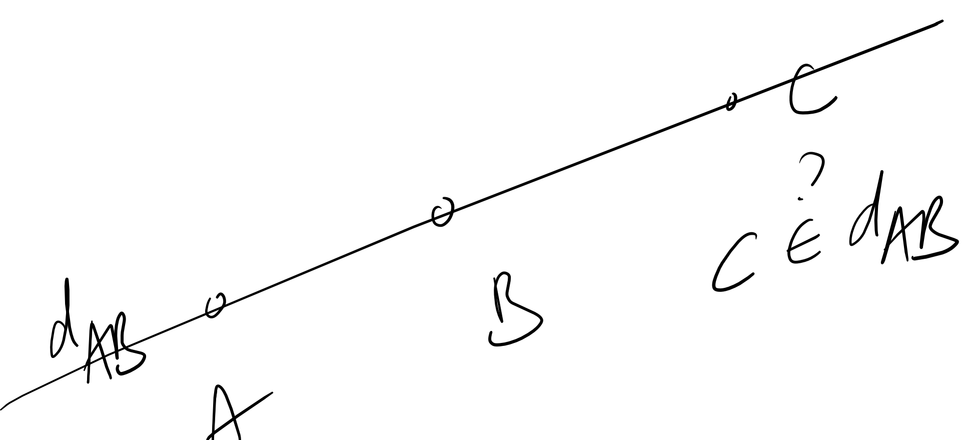


$$\overrightarrow{OM'} = \frac{1}{2}\overrightarrow{OM}$$

$\Rightarrow O, M, M'$ sont alignés

$$M' \overset{?}{\in} d_{OM} : 3\cdot\left(-\frac{1}{2}\right)+\frac{3}{2} = -\frac{3}{2}+\frac{3}{2} = 0$$

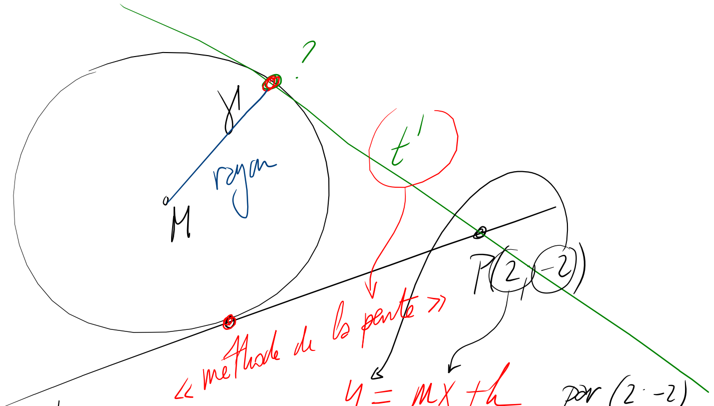
$\Rightarrow$ Oui



$$C \overset{?}{\in} d_{AB}$$

f)

$\boxed{P \in t_B:}$



« méthode de la pente »

$4x - y - 10 = 0$

$4 \cdot 2 - (-2) - 10 = 0$

$8 + 2 - 10 = 0$

$0 = 0 \Rightarrow$ Oui

$y = mx + h$ par $(2; -2)$

pente, o.o.

$t': mx - y + h = 0$

$M(-1; 3)$

$-2 = 2m + h$

$h = -2m - 2$

$\text{dist}(M, t') = \text{rayon} = \sqrt{17}$

$\text{dist}((x_0; y_0); ax + by + c) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

$$\frac{|m \cdot (-1) - 3 + h|}{\sqrt{m^2+1}} = \sqrt{17}$$

$$\frac{|-m - 3 - 2m - 2|}{\sqrt{m^2+1}} = \sqrt{17}$$

$$\frac{|-(3m+5)|}{\sqrt{m^2+1}} = \sqrt{17} \qquad \frac{(3m+5)^2}{m^2+1} = 17$$

$$\frac{9m^2 + 30m + 25}{m^2+1} = 17$$

$9m^2 + 30m + 25 = 17m^2 + 17$

$8m^2 - 30m - 8 = 0$

$4m^2 - 15m - 4 = 0$

$$m = \frac{15 \pm \sqrt{289}}{8} = \frac{15 \pm 17}{8} \begin{cases} 4 \\ -1/4 \end{cases}$$

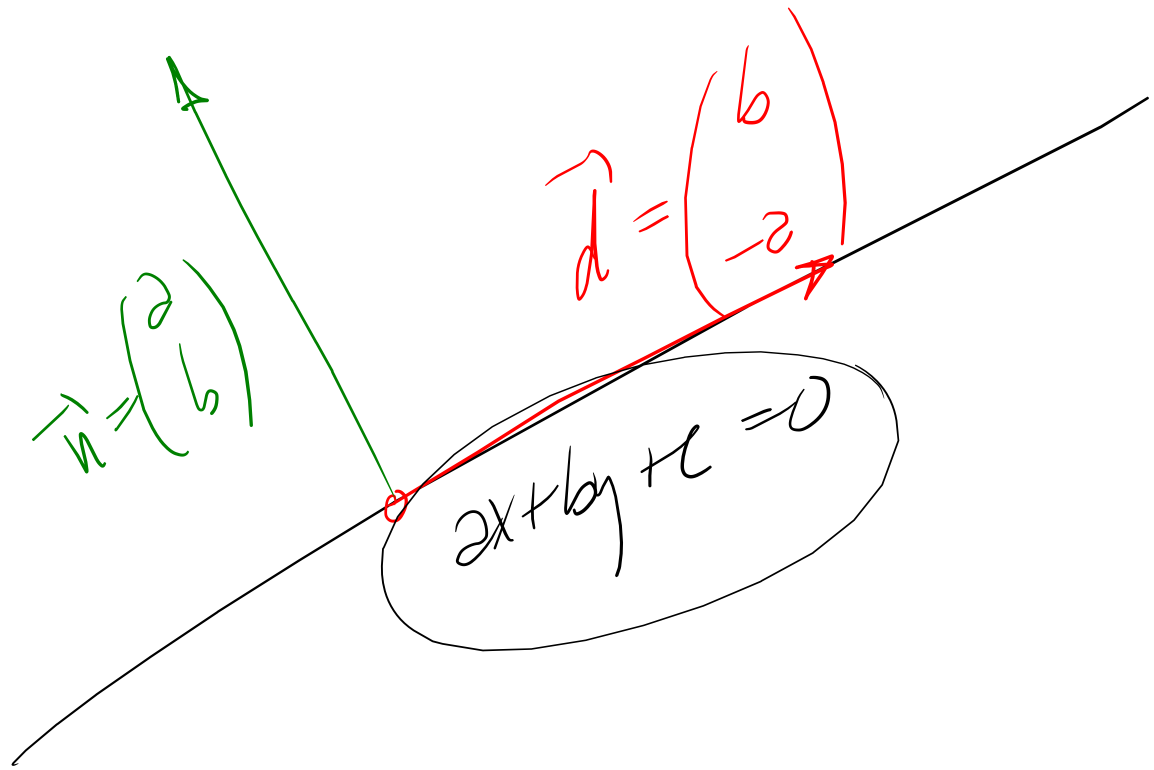
$y = 4x - 2 \cdot 4 - 2$

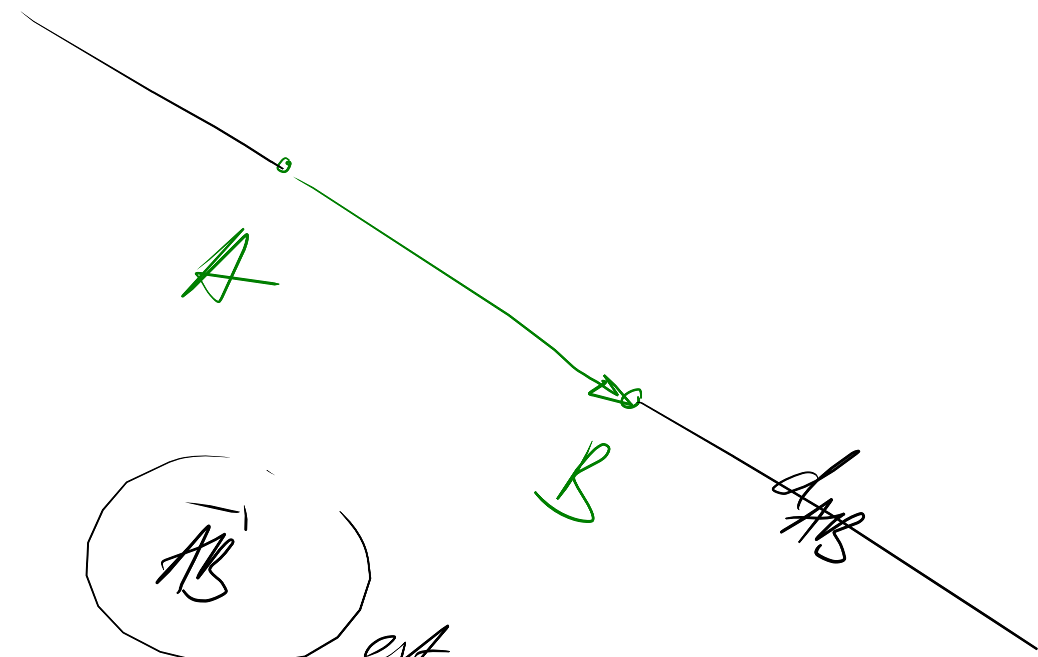
$t: y = 4x - 10$

$t': y = -\frac{1}{4}x + \frac{1}{2} - 2$

$y = -\frac{1}{4}x - \frac{3}{2}$

$\boxed{t': x + 4y + 6 = 0}$





$\vec{AB}$ est directeur de d_{AB}