$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b \qquad F'(x) = f(x)}$$

INTÉGRALE DÉFINIE

$$\boxed{\int f(x)\,dx = F(x) + c \iff F'(x) = f(x)}$$

PRIMITIVE

APPLICATIONS : AIRES / VOLUMES

P4 / P5 2023

P4 / P5 2022

P4 2023

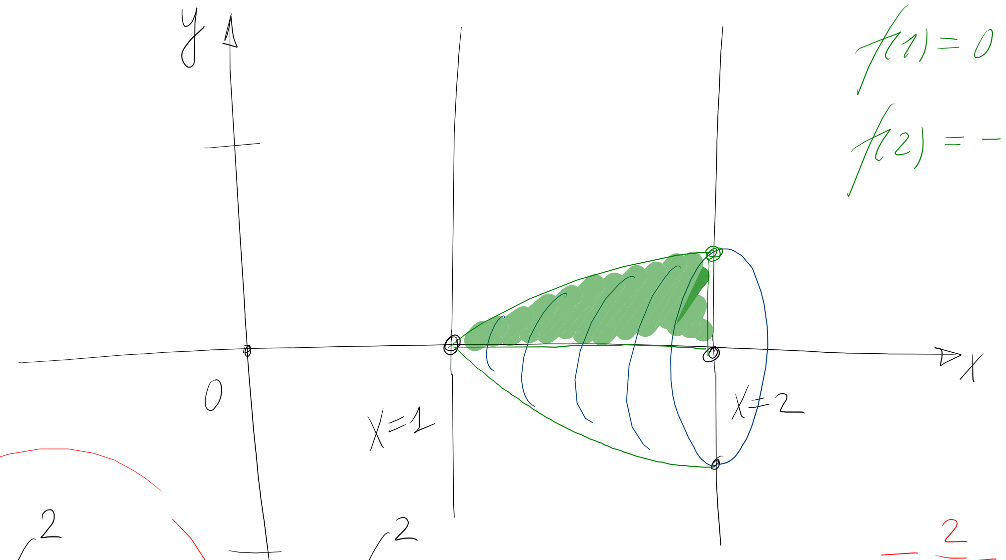
Montrer que

a) $$\left(f(x)\right)^2 = \left(\frac{1-x}{x}\right)^2 = \frac{(1-x)^2}{x^2} = \frac{1-2x+x^2}{x^2}$$

$$= \frac{1}{x^2} - \frac{2x}{x^2} + \frac{\cancel{x^2}\ 1}{\cancel{x^2}\ 1} = \boxed{\frac{1}{x^2} - \frac{2}{x} + 1}$$

$$\frac{2\cdot\cancel{x}}{x\cdot\cancel{x}}$$

b)



$f(1) = 0$

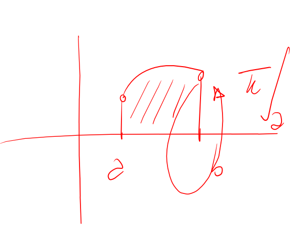
$f(2) = -\frac{1}{2}$

$$\pi \int_1^2 f^2(x)\,dx = \pi \int_1^2 \left(\frac{1}{x^2} - \frac{2}{x} + 1\right) dx$$

$-\frac{2}{x} = -2 \cdot \frac{1}{x}$

$= -2 \cdot x^{-1}$

$$= \pi \int_1^2 x^{-2}\,dx - 2\pi \int_1^2 x^{-1}\,dx + \pi \int_1^2 1\,dx$$



$\pi \int_a^b f^2(x)\,dx$

$-\pi \int_1^2 2 \cdot x^{-1}\,dx$

$\int x^n\,dx = \frac{1}{n+1} \cdot x^{n+1} + C$

si $n \neq -1$

$\int x^{-1}\,dx = \ln|x| + C$

$$= \pi \underbrace{\frac{1}{1+(-2)}}_{(-1)\,\cdot\, x^{-1}} \cdot \underbrace{x^{1+(-2)}}_{x^{-1}} \Big|_1^2 - 2\pi \cdot \ln|x| \Big|_1^2 + \pi \cdot x \Big|_1^2$$

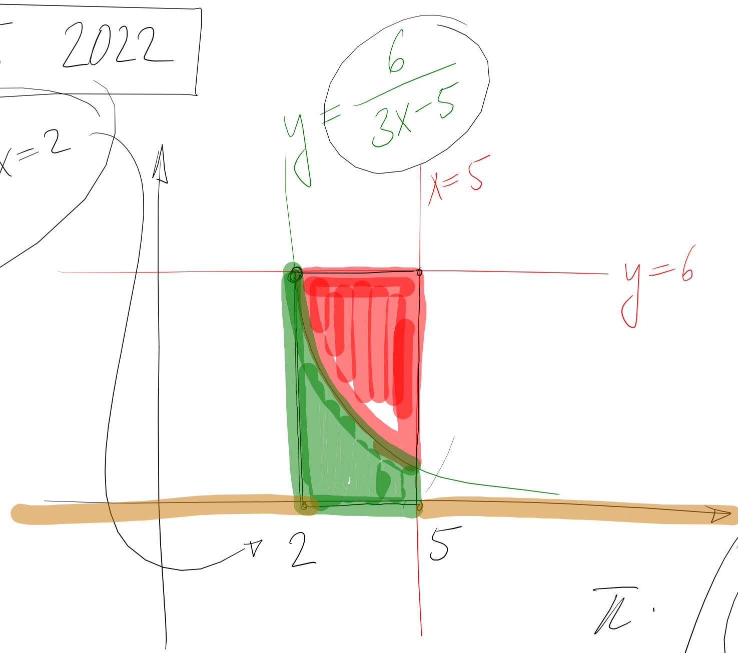
$$= \pi \cdot (-1) \cdot \left(2^{-1} - 1^{-1}\right) - 2\pi \left(\ln 2 - \underbrace{\ln 1}_{=0}\right) + \pi \cdot (2-1)$$

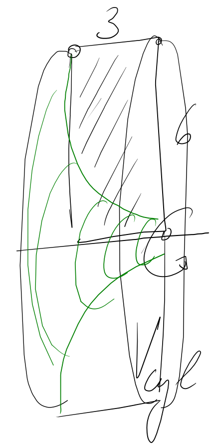
$$= -\pi \cdot \left(\frac{1}{2} - 1\right) - 2\pi \ln 2 + \pi$$

$$= \frac{\pi}{2} + \pi - 2\pi \ln 2 = \frac{3\pi}{2} - 2\pi \ln 2 \simeq 0{,}35 > 0$$

P5 2022

$$6 = \frac{6}{3x-5} \Rightarrow x=2$$





$$V_{cyl} = 108\pi$$

$$\pi \cdot \int_2^5 \left(\frac{6}{3x-5}\right)^2 dx =$$

$$\pi \int_2^5 \frac{36}{(3x-5)^2}\, dx =$$

$$\int \frac{1}{t^2}\,dt = -\frac{1}{t} + c$$

$$12\pi \int_2^5 \frac{1}{(3x-5)^2} \cdot 3 \cdot dx = 12\pi \cdot \int_2^5 \frac{1}{(3x-5)^2} \cdot (3x-5)' \cdot dx =$$

$$12\pi\left(\frac{-1}{3x-5}\right)\Bigg|_2^5 = -12\pi \cdot \left(\frac{1}{15-5} - \frac{1}{6-5}\right)$$

$$\frac{1}{t^2} = t^{-2}$$

$$= -12\pi\left(\frac{1}{10} - \frac{1}{1}\right) = -12\pi \cdot \left(-\frac{9}{10}\right)$$

$$\int t^{-2}\,dt = \frac{1}{-2+1} \cdot t^{-2+1} + c$$

$$= -1 \cdot t^{-1} + c$$

$$= \frac{108\pi}{10}$$

Volume

$$\Rightarrow V = \boxed{108\pi - \frac{108\pi}{10}} = (108 - 10{,}8)\pi = 97{,}2\pi$$

$$= \left(\frac{1080-108}{10}\right)\pi = \frac{972}{10} \cdot \pi$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = « \frac{0}{0} » \quad \text{IND} \quad « \frac{\infty}{\infty} »$$

Calcul à faire

et que $$\lim_{x \to a} \frac{f'(x)}{g'(x)} = C$$

$$\lim_{x \to \infty} \frac{x^3 - 1}{x^3 + 1}$$

$$\Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

$$\int x\sqrt{9-x^2}\;dx$$

$$\frac{1}{-2}\int \sqrt{9-x^2}\cdot(-2x)\cdot dx$$

$$(\;)'$$

$$-\frac{1}{2}\cdot\frac{2}{3}\sqrt{(9-x^2)^3}$$

$$\int\sqrt{t}\;dt=\int t^{1/2}\,dt$$

$$=\frac{1}{1+\frac{1}{2}}\cdot t^{1+\frac{1}{2}}$$

$$=\frac{2}{3}\,t^{\frac{3}{2}}$$

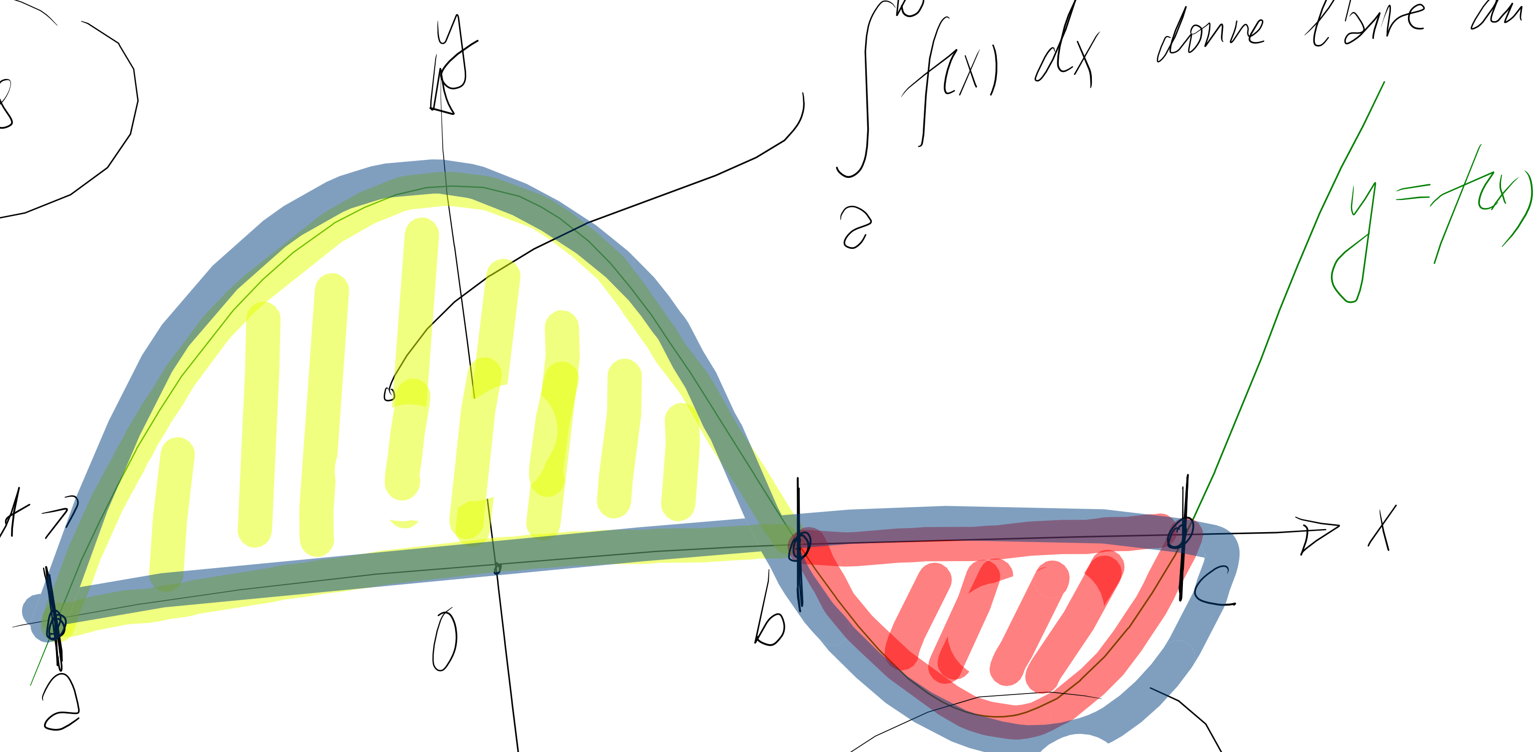
$$=\frac{2}{3}\sqrt{t^3}$$

Aires

$\int_a^b f(x)\,dx$ donne l'aire du domaine

car $f(x) \geq 0$

si $a \leq x \leq b$



$\int_a^c f(x)\,dx$ soustrait « automatiquement » les deux aires.

$-\int_b^c f(x)\,dx$ donne l'aire

car $f(x) \leq 0$

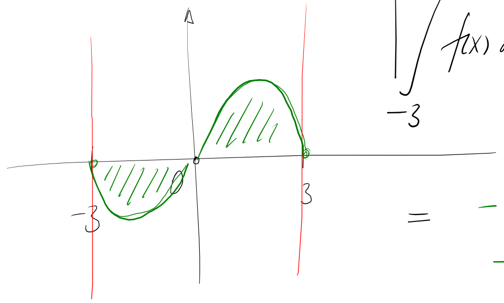
si $b \leq x \leq c$

$\boxed{\text{P5 juin 2023}}$ $\boxed{f(x) = x\cdot\sqrt{(3+x)(3-x)}}$

a) $f(x)=0 \Leftrightarrow \boxed{\begin{matrix} x=0 \\ x=-3 \\ x=3 \end{matrix}}$

$9-x^2 \geq 0 \Leftrightarrow x\in[-3;3]$

b)



$$\left|\int_{-3}^{0} f(x)\,dx\right| + \left|\int_{0}^{3} f(x)\,dx\right| =$$

$$= -\int_{-3}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx$$

$$\int f(x)\,dx = \int x\sqrt{9-x^2}\,dx = -\frac{1}{2}\int \sqrt{9-x^2}\cdot(-2x)\,dx$$

$?\,(\,)'$

$$= \int (9-x^2)^{\frac{1}{2}}\cdot(9-x^2)'\,dx = \left(-\frac{1}{2}\right)\frac{1}{1+\frac{1}{2}}(9-x^2)^{1+\frac{1}{2}} + C$$

$$= \left(-\frac{1}{2}\right)\frac{2}{3}(9-x^2)^{\frac{3}{2}} + C = -\frac{1}{3}(9-x^2)^{\frac{3}{2}} + C$$

Aire :

$$\left|\left[-\frac{1}{3}(9-x^2)^{\frac{3}{2}}\right]_{-3}^{0}\right| + \left|\left[-\frac{1}{3}(9-x^2)^{\frac{3}{2}}\right]_{0}^{3}\right|$$

$$\underbrace{\left|\left(-\frac{1}{3}\cdot 9^{\frac{3}{2}}\right) - \left(-\frac{1}{3}\right)\Big(\underbrace{9-(-3)^2}_{=0}\Big)^{\frac{3}{2}}\right|}_{|-9|} + \underbrace{\left|\left(-\frac{1}{3}\,\big(\underbrace{9-9}_{0}\big)^{\frac{3}{2}}\right) - \left(-\frac{1}{3}\right)(9)^{\frac{3}{2}}\right|}_{|9|}$$

$$|-9| + |9| = 18$$