1) $$\int \frac{1}{x+1}\,dx = \int \frac{1}{x+1}\cdot 1\cdot dx$$

$$= \int \frac{1}{x+1}\cdot (x+1)'\,dx$$

$$= \ln|x+1| + C$$

2) $$\int \frac{1}{2x+3}\,dx = \int \frac{1}{2x+3}\cdot 2\cdot \frac{1}{2}\cdot dx$$

$$= \frac{1}{2}\int \frac{1}{2x+3}\cdot 2\,dx$$

$$= \frac{1}{2}\int \frac{1}{2x+3}\cdot (2x+3)'\,dx$$

$$= \frac{1}{2}\cdot \ln|2x+3| + C$$

3) $\int \frac{1}{x^2-2x+4} \cdot (x-1)\,dx =$

$\int \frac{1}{x^2-2x+4} \cdot (2x-2) \cdot \frac{1}{2} \cdot dx =$

$\frac{1}{2} \int \frac{1}{x^2-2x+4} \cdot (x^2-2x+4)' \,dx =$

$\frac{1}{2} \ln |x^2-2x+4| + C$

4) $\int \left(x^2+1+\frac{3}{5x-1}\right) dx =$

$\int (x^2+1)\,dx + \int \frac{3}{5x-1}\,dx =$

$\frac{1}{3}x^3 + x + 3\int \frac{1}{5x-1} \cdot 5 \cdot \frac{1}{5}\,dx =$

$$\frac{1}{3}x^3+x+3\cdot\frac{1}{5}\int\frac{1}{5x-2}\cdot(5x-2)'\,dx=$$

$$\frac{1}{3}x^3+x+\frac{3}{5}\cdot\ln|5x-1|+C$$

5) ⚠ degré(x^2+2x-2) > degré$(x-1)$

$\Rightarrow$ On effectue la division euclidienne

$$\begin{array}{r|l} x^2+2x-2 & x-1 \\ \cline{2-2} x^2-x & x+3 \\ \cline{1-1} 3x-2 & \\ 3x-3 & \\ \cline{1-1} 1 & \end{array}$$

$$\Rightarrow\quad x^2+2x-2=(x-1)(x+3)+1$$

$$\Rightarrow\quad \frac{x^2+2x-2}{x-1}=\frac{(x-1)(x+3)+1}{x-1}=x+3+\frac{1}{x-1}$$

On peut alors écrire :

$$\int \frac{x^2+2x-2}{x-1}\,dx = \int \left(x+3+\frac{1}{x-1}\right)dx$$

$$= \int (x+3)\,dx + \int \frac{1}{x-1}\cdot (x-1)'\cdot dx$$

$$= \frac{1}{2}x^2+3x+\ln|x-1|+C$$

6)

$$\begin{array}{r|l} 3x^3+6x^2-3x+5 & x+2 \\ \cline{2-2} 3x^3+6x^2 & 3x^2-3 \\ \cline{1-1} \quad -3x+5 & \\ \quad -3x-6 & \\ \cline{1-1} \quad 11 & \end{array}$$

$$\frac{3x^3+6x^2-3x+5}{x+2} = \frac{(x+2)(3x^2-3)}{x+2} + \frac{11}{x+2}$$

On peut écrire:

$$\int \frac{3x^3+6x^2-3x+5}{x+2}\,dx = \int (3x^2-3)\,dx + \int \frac{11}{x+2}\,dx$$

$$= 3\cdot\frac{1}{3}x^3 - 3x + 11\cdot\int \frac{1}{x+2}\cdot(x+2)'\,dx$$

$$= x^3 - 3x + 11\ln|x+2| + C$$

7) $$\int \frac{1}{\sin x}\cdot\cos x\,dx =$$

$$\int \frac{1}{\sin x}\cdot(\sin x)'\,dx = \ln|\sin x| + C$$

$$8) \int \operatorname{tg} x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$$= \int \frac{1}{\cos x} \cdot \sin x \cdot (-1) \cdot (-1) \, dx$$

$$= -\int \frac{1}{\cos x} (-\sin x) \, dx$$

$$= -\int \frac{1}{\cos x} \cdot (\cos x)' \, dx$$

$$= -\ln|\cos x| + C$$