Intégration des fractions rationnelles

$P(x)$, $Q(x)$ deux polynômes

But: Calculer $\displaystyle\int \frac{P(x)}{Q(x)}\,dx$

Exemple: $\displaystyle\int \frac{x^2-2x+3}{x^3+1}\,dx = \,?$

Intro:

$$\frac{1}{12} = \frac{1}{3\cdot \boxed{2\cdot 2}} = \frac{a}{3} + \frac{b}{2} + \frac{c}{2^2} = \frac{1}{3} - \frac{1}{4}$$

$$\boxed{\frac{1}{x^2+4x+3} = \frac{1}{(x+1)(x+3)} =}$$

$$\frac{1}{12} = \frac{a}{3} + \frac{b}{2} + \frac{c}{4} = \frac{4a+6b+3c}{12}$$

$1 = 4a + 6b + 3c$

$c = -1$

$b = 0$

$1 = 4 \cdot 1 + 6 \cdot 0 + 3 \cdot (-1)$

$a = 1$

Calculer $\displaystyle\int \frac{1}{x^2+4x+3}\,dx = \int \frac{1}{(x+1)(x+3)}\,dx$

$$\boxed{\frac{1}{(x+1)(x+3)} = \frac{a}{x+1} + \frac{b}{x+3}}$$

$a = 1/2$, $b = -1/2$

$a, b \in \mathbb{R}$

$\forall x \in \mathbb{R}$

0 si $x = -3$

$$\frac{1}{(x+1)} = \frac{a(x+3)}{x+1} + b \qquad \Big/ \qquad x = -3$$

$$\frac{1}{-2} = b \qquad \Big/ \qquad b = -\frac{1}{2}$$

$$\frac{1}{x+3} = a + \frac{b(x+1)}{x+3} \qquad \Big/ \qquad x = -1$$

$$\frac{1}{2} = a$$

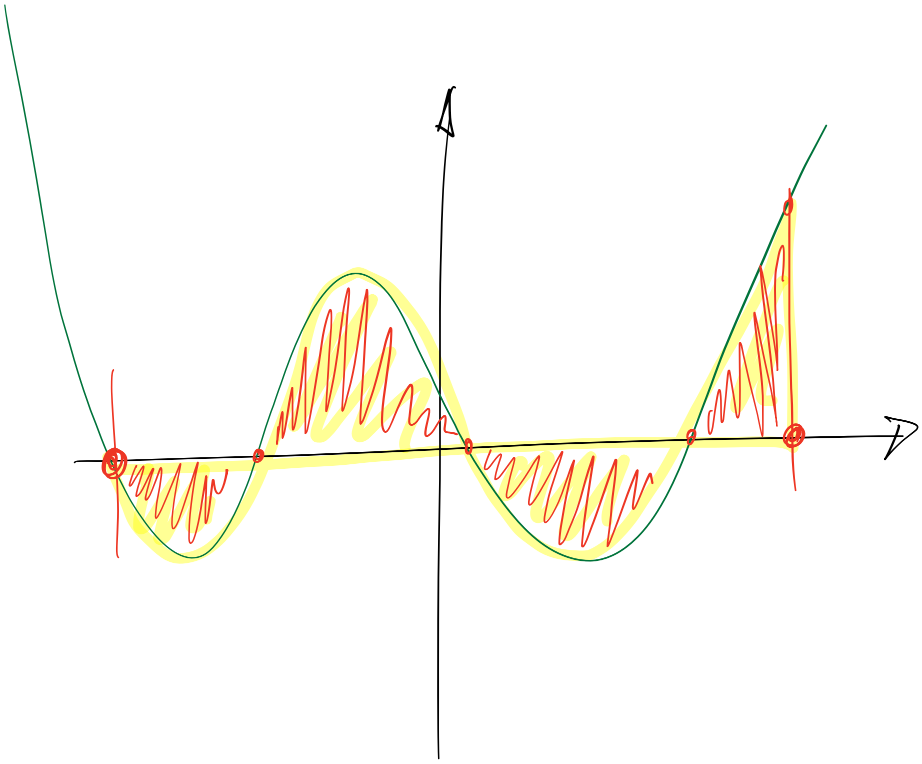
$$\int \frac{1}{x^2+4x+3}\,dx = \frac{1}{2}\int \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{1}{x+3}\,dx$$

$$= \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3) + C$$

$$\frac{1/2\,(x+3)}{x+1\,(x+3)} + \frac{(-1/2)\,(x+1)}{(x+3)\,(x+1)} = \frac{\frac{1}{2}(x+3) - \frac{1}{2}(x+1)}{(x+1)(x+3)}$$

$$= \frac{\cancel{\frac{1}{2}x} + \frac{3}{2} - \cancel{\frac{1}{2}x} - \frac{1}{2}}{(x+1)(x+3)}$$

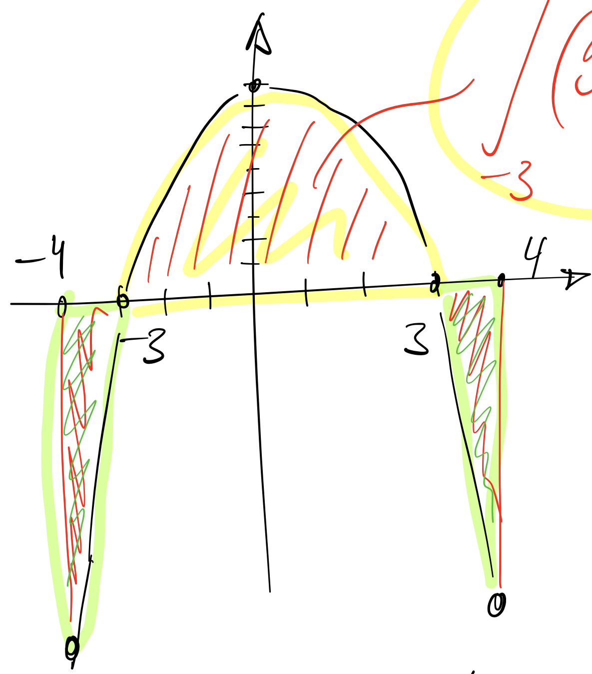
$$= \frac{1}{(x+1)(x+3)}$$



$f(x) = 9 - x^2$

$= (3-x)(3+x)$

$\int_{-3}^{3} (9-x^2)\,dx$



$9x - \frac{1}{3}x^3 = \int f(x)\,dx$

$$\underbrace{\left|\int_{-4}^{-3} f(x)\,dx\right|}_{A_1} + \underbrace{\left|\int_{-3}^{3} f(x)\,dx\right|}_{A_2} + \underbrace{\left|\int_{3}^{4} f(x)\,dx\right|}_{A_1} = A$$

$$A = 2 \cdot A_1 + A_2$$

$$A_1 = \left| \left(9x - \frac{1}{3}x^3\right)\Big|_{-4}^{-3} \right|$$

$$= \left| \left(-27 - \frac{1}{3}(-27)\right) - \left(-36 - \frac{1}{3}(-64)\right) \right|$$

$+9$

$$= \left| -18 + 36 - \frac{64}{3} \right| = \left| 18 - \frac{64}{3} \right| = \left| \frac{54 - 64}{3} \right|$$

$$= \frac{10}{3}$$