$\boxed{6.3}$

$$\boxed{H = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}}$$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "H"] & \mathbb{R}^2 \arrow[d, "P^{-1}"] \\ \mathbb{R}^2 \arrow[u, "P"] \arrow[r, "H'"] & \mathbb{R}^2 \end{tikzcd}$$

$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto H'\begin{pmatrix} x \\ y \end{pmatrix}$

$P^{-1} \cdot H \cdot P \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

$\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$ $\quad x\begin{pmatrix} 1 \\ 2 \end{pmatrix} + y\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ $\quad P = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

$$\boxed{H' = P^{-1} H P}$$

$$\underbrace{\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)}_{P} \longrightarrow \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{I_2} \left| \begin{pmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{pmatrix} \right. P^{-1}$$

$L_2 \leftarrow L_2 - 2L_1$

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right)$$

$L_2 \leftarrow \frac{1}{5} L_2$

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2/5 & 1/5 \end{array}\right)$$

$L_1 \leftarrow L_1 + 2L_2$

$$\Rightarrow P^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

$P \cdot P^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$P^{-1} \cdot P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \frac{1}{5} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underbrace{P^{-1}}\cdot H \cdot P$$

$$H' = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 7 & 1 \\ 6 & 8 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 19 & 17 \\ -8 & 6 \end{pmatrix} = H'$$

$H = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$ matrice de $h : \mathbb{R}^2 \to \mathbb{R}^2$

$(x, y) \longmapsto (x + 3y, -2x + 4y)$

$\ker H = \ker h \longrightarrow$ ens des sols de

$$\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 10 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$10x_2 = 0 \Rightarrow x_2 = 0$

$x_1 + 3x_2 = 0 \Rightarrow x_1 = 0$

$\Rightarrow \ker H = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

$U \xrightarrow{h} V$ $U, V$ espaces vectoriels de dim finie

$$\boxed{\dim U = \dim \ker(h) + \dim \operatorname{Im}(h)}$$

Théorème du rang.

$\ker(h)$ : $h(x) = 0$ à résoudre

$\operatorname{Im}(h)$ : Trouver la base (cf. technique pour les matrices)

Exemple: $h$ donné par $H = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

$\mathbb{R}^3 \xrightarrow{h} \mathbb{R}^3$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto h\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = H \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$\ker h$ : $\begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ à résoudre

$\operatorname{Im} h$ : Les colonnes pivots dans la matrice éch. réduite donnent les colonnes de $H$ à considérer.

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 5 & -1 \\ 0 & 4 & 1 \end{matrix} \sim \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 4 & 1 \end{matrix} \sim \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 9 \end{matrix}$$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{matrix}$$

$U \xrightarrow{h} U$ $\quad$ $U$ esp. vect. de dim. finie

$h$ linéaire ($h$ endomorphisme)

$h(\alpha x + \beta y) = \alpha h(x) + \beta h(y)$

$h(0) = 0$

$h(u) = H \cdot u$

matrice carrée

$$\boxed{h \text{ injectif} \Leftrightarrow h \text{ surjectif} \Leftrightarrow h \text{ bijectif}}$$

Cas particulier des applications linéaires

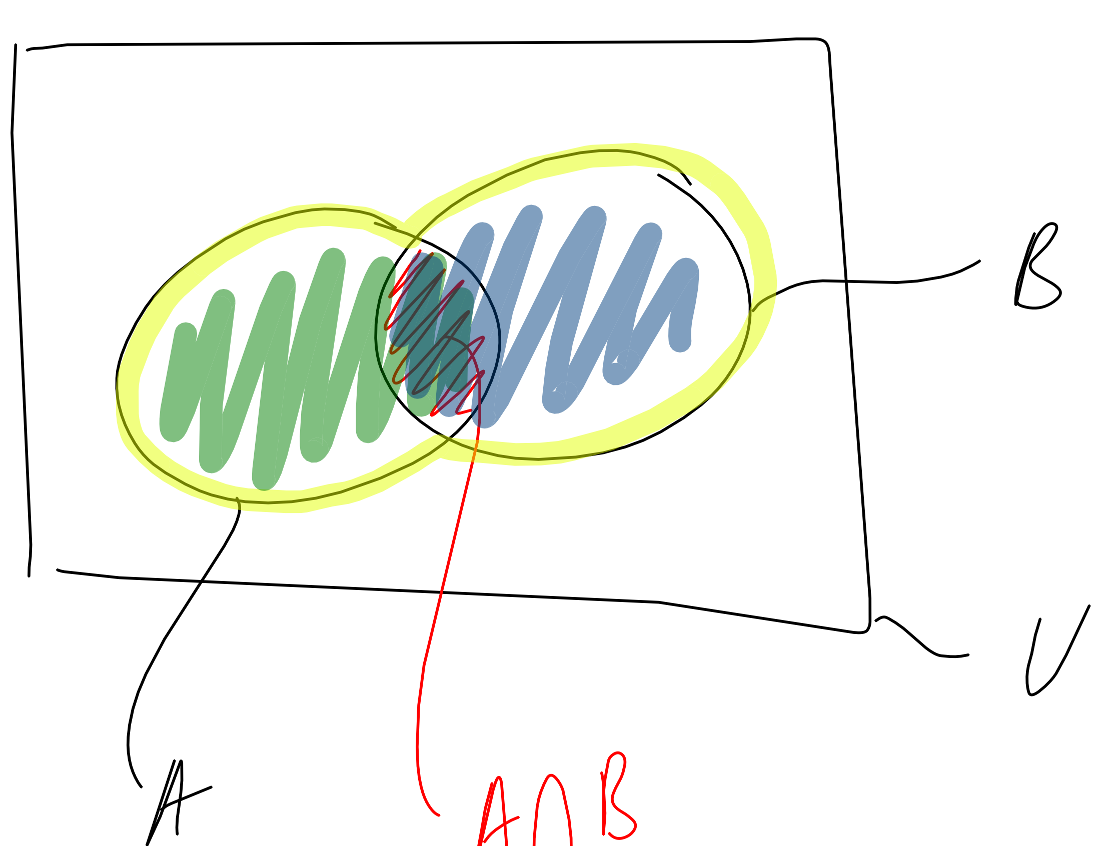
$h$ injectif $\Leftrightarrow$ $\ker h = \{0\}$

$h$ surjectif $\Leftrightarrow$ $\operatorname{Im} h = U$

Thm du rang

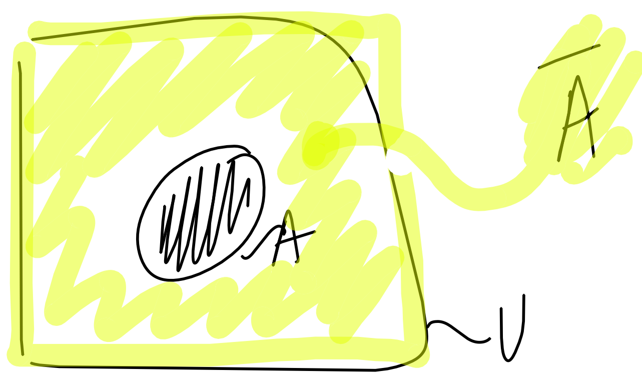
$\dim(U) = n \implies n = \dim(\ker h) + \dim(\operatorname{Im} h)$

$U \xrightarrow{h} U$

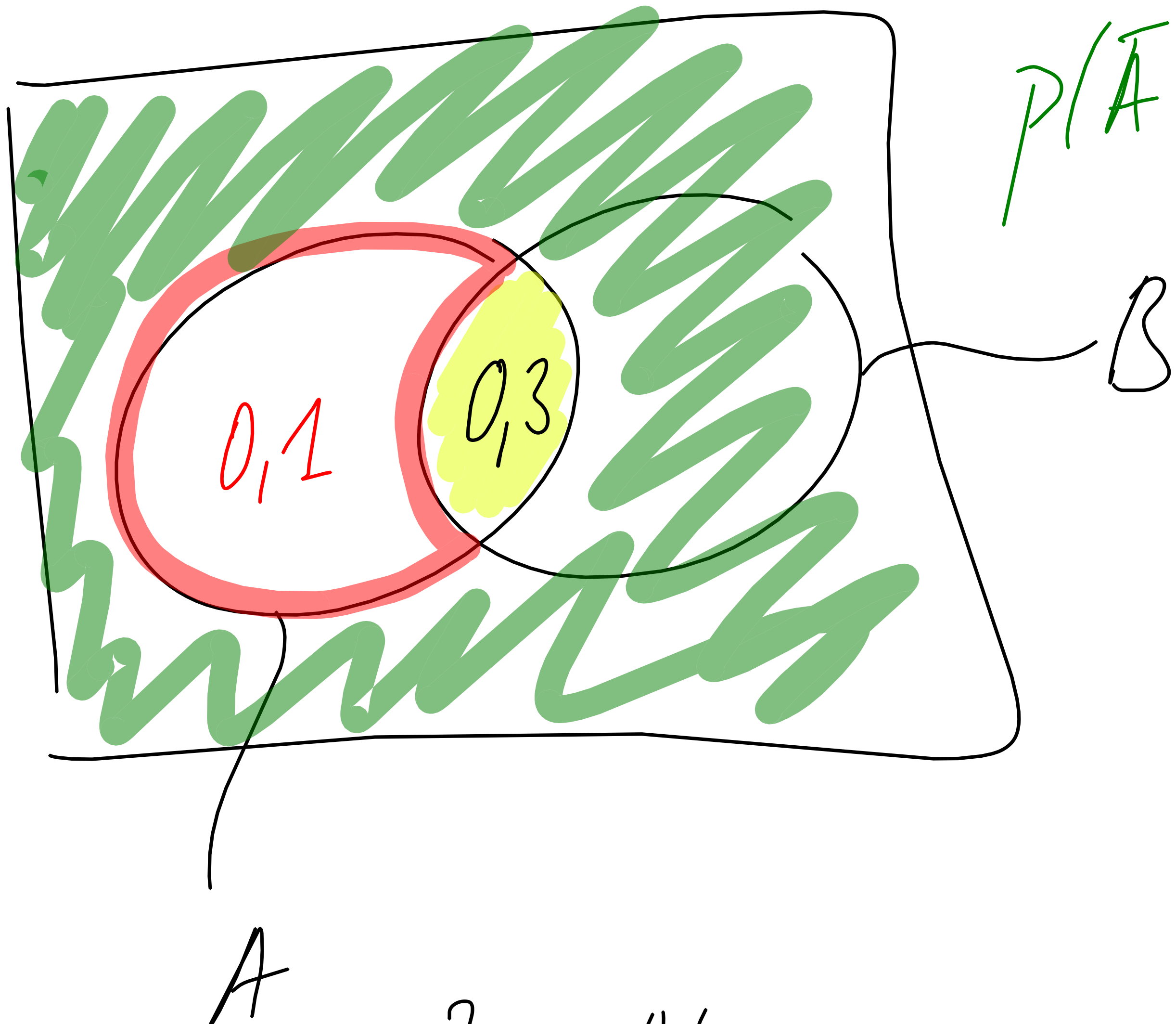



$p(U) = 1$

$$\boxed{p(A \cup B) = p(A) + p(B) - p(A \cap B)}$$




$\bar{A} = \complement_U A$ complementaire




$P(\bar{A}) = 0{,}6$

$P(B) = \frac{1}{2} = 0{,}5$

$P(A \cap B) = \frac{3}{10} = 0{,}3$

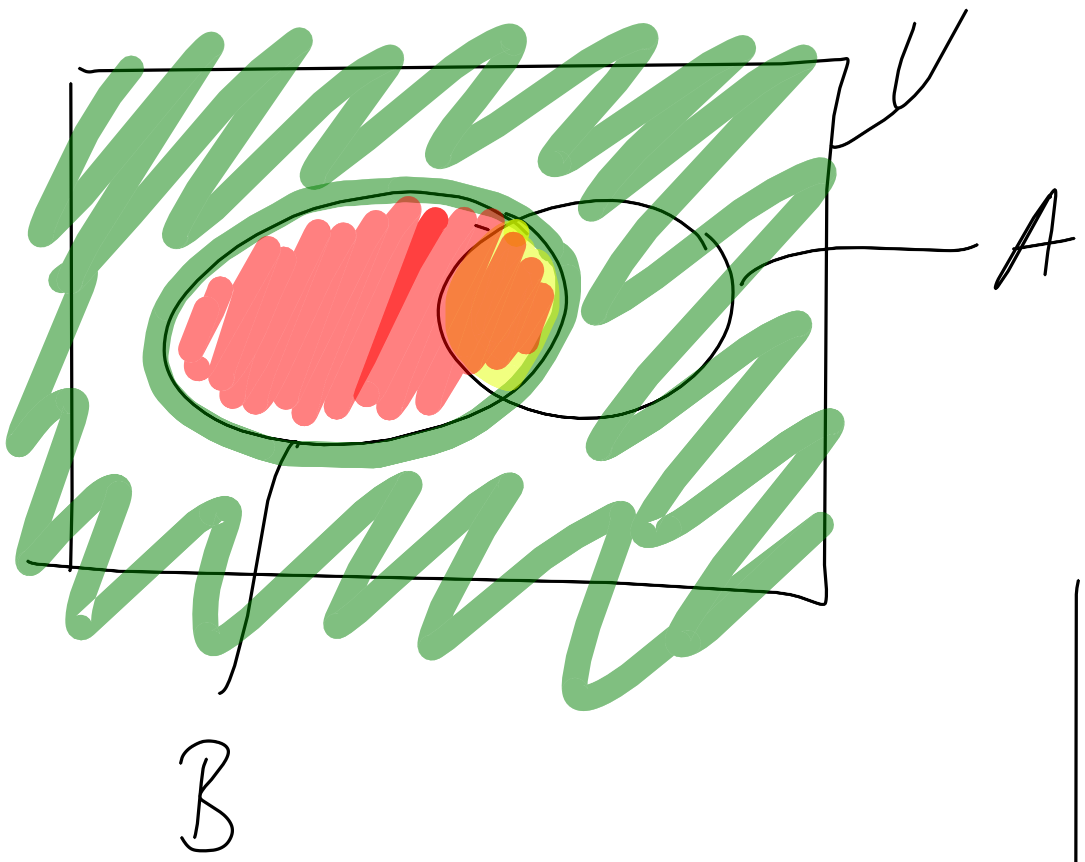
$P(A) = \frac{2}{5} = 0{,}4$

Tous les exos du § 4.2 sauf

4.2.13

4.2.19

# Probas conditionnelles




## Bayes

B a eu lieu  et

$$p(A|B) = \frac{p(A\cap B)}{p(B)}$$

Sachant que

A, B indépendants

$$p(A|B) = p(A)$$

$$\Rightarrow \quad p(A\cap B) = p(A)\cdot p(B)$$

| | | | | | |
|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 |
| 21 | 22 | 23 | 24 | 25 | 26 |
| 31 | 32 | 33 | 34 | 35 | 36 |
| 41 | 42 | 43 | 44 | 45 | 46 |
| 51 | 52 | 53 | 54 | 55 | 56 |
| 61 | 62 | 63 | 64 | 65 | 66 |

$\Sigma = 7$

$$p(A|B) = \boxed{p(\Sigma=7 \mid \text{„5 an 5“}) = \frac{2/36 \;\; p(A \cap B)}{11/36 \;\; p(B)}}$$

$$p(\Sigma = 7) = \frac{1}{6} = \frac{6}{36}$$

$$p(\text{„5 an 5“}) = \frac{11}{36}$$

A: $\Sigma = 7$

B: „5 an 5“

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= \frac{6}{36} + \frac{11}{36} - \frac{2}{36}$$