Méthodes de calcul de primitives

1. Tables

2. $\int f(g(x)) \cdot g'(x)\,dx = F(g(x)) + c$

   avec $F' = f$

3. Par parties

Example ②

$$\int \sqrt{x^2+1} \cdot x \, dx$$

$$(x^2+1)' = 2x$$

$$= \int \sqrt{x^2+1} \cdot \underbrace{\frac{1}{2} \cdot 2}_{1} x \, dx$$

$$= \int \frac{1}{2} \sqrt{x^2+1} \cdot 2x \, dx$$

$$= \frac{1}{2} \int \sqrt{\boxed{x^2+1}} \cdot 2x \, dx = \frac{1}{2} \cdot \frac{2}{3} \cdot (x^2+1)^{\frac{3}{2}} + C$$

$$(\;)'$$

$$\int \sqrt{T} \, dT = \frac{2}{3} T^{\frac{3}{2}} + C$$

$$\left(\frac{2}{3} T^{\frac{3}{2}}\right)' = \frac{2}{3} \cdot \frac{3}{2} \cdot T^{\frac{3}{2}-\frac{2}{2}}$$

$$= 1 \cdot T^{\frac{1}{2}}$$

$$= \sqrt{T} \checkmark$$

$$\left(\frac{1}{2} \cdot \frac{2}{3} \cdot (x^2+1)^{\frac{3}{2}}\right)' = \frac{1}{3} \cdot \frac{3}{2} \cdot (x^2+1)^{\frac{3}{2}-1} \cdot 2x$$

$$= (x^2+1)^{\frac{1}{2}} \cdot x$$

$$= \sqrt{x^2+1} \cdot x$$

$$\boxed{\int x^2 \, dx = \frac{1}{2+1} \cdot x^{2+1} + C}$$

$$(x^n)' = n x^{n-1}$$

$$2^{\frac{p}{q}} = \sqrt[q]{2^p}$$

$\sqrt[2]{x^1}$

$x^{\frac{1}{2}}$

$\frac{1}{1+1/2} \cdot x^{\frac{1}{2}+1} + C$

$\frac{2}{3} \cdot \sqrt{x^3} + C$

$\frac{2}{3} x^{\frac{3}{2}} + C$

$$\int \boxed{\cos(3x)}\,dx =$$

$(3x)' = 3$

$$\int \cos(T)\,dT \overset{\text{table}}{=} \sin(T) + C$$

$$\frac{1}{3}\int \cos(3x)\, 3 \cdot dx = \frac{1}{3}\int \cos(3x) \cdot (3x)' \cdot dx$$

$f(g(x)) \cdot g'(x)$

$$= \frac{1}{3}\left(\sin(3x)\right) + C$$

$$= \frac{1}{3}\sin(3x) + C$$

$$\cancel{\int \sin^2 x \cdot \cos x \, dx = \underbrace{\tfrac{1}{2}\left(x - \sin x \cos x\right)}_{\int \sin^2 x \, dx} \cdot \underbrace{(-\sin x)}_{\int \cos x \, dx} + C}$$

$$\cancel{\int f \cdot g = \int f \cdot \int g}$$

$$\int (\overbrace{\sin x}^{T})^2 \cdot \overbrace{\cos x \, dx}^{dT} = \int T^2 \, dT$$

$$(\sin x)' = \cos x$$

si $T = \sin x$

$$= \frac{1}{3} T^3 + C$$

$$= \frac{1}{3} (\sin x)^3 + C$$

$$(f \cdot g)' = f'g + fg'$$

$$\int (f \cdot g)' = \int (f'g + fg')$$

$$= \int f'g + \int fg'$$

$$f \cdot g = \int f'g + \int fg'$$

$$\int f'g = f \cdot g - \int fg'$$

$$\int f'(x) \cdot g(x)\, dx = f(x) \cdot g(x) - \int f(x)\, g'(x)\, dx$$

$$\int \cos x \cdot \cos x\, dx = \sin x \cos x - \int \sin x\, (-\sin x)\, dx$$

$$= \sin x \cos x + \int \underbrace{\sin^2 x}_{\int (1 - \cos^2 x)\, dx}\, dx$$

$$\int \cos^2 x\, dx = \sin x \cos x + \int 1\, dx - \int \cos^2 x\, dx$$

$$2 \int \cos^2 x\, dx = x + \sin x \cos x + C$$

$$\int \cos^2 x\, dx = \frac{1}{2}(x + \sin x \cos x) + C$$

$\sin(2x) = 2 \sin x \cos x$

$$\frac{x}{2} + \frac{\sin 2x}{4} = \frac{x}{2} + \frac{2 \sin x \cos x}{4} \checkmark$$

$$(f \cdot g)' = f g' + f' g$$

$$\underbrace{\left(x \cdot (-\cos x)\right)'}_{(f \cdot g)'} = \underbrace{\boxed{x \cdot \sin x}}_{f \cdot g'} + \underbrace{1 \cdot (-\cos x)}_{f' \cdot g}$$

$$\int x \sin x \, dx = x \cdot (-\cos x) - \int -\cos x \, dx$$

$$= -x \cos x + \sin x + C$$

$$\frac{1}{\sqrt{2}} = \frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}}$$

$$\frac{x+1}{\sqrt{x^2+2x}} = \left(x^2+2x\right)^{-\frac{1}{2}} \cdot (x+1)$$

$$= \frac{1}{2}\left(x^2+2x\right)^{-\frac{1}{2}}(2x+2)$$

$$\left(x^2+2x\right)' = 2x+2$$

$$T = x^2+2x$$

$$(2x+2)\,dx = dT$$

$$\Rightarrow \int \frac{x+1}{\sqrt{x^2+2x}}\,dx = \frac{1}{2}\int T^{-\frac{1}{2}}\,dT$$

$n \in \mathbb{Q}$ $\quad \int x^n\,dx = \frac{1}{1+n}x^{n+1} + C$

$$= \frac{1}{2}\int \left(x^2+2x\right)^{-\frac{1}{2}} \cdot (2x+2)\,dx = \frac{1}{2} \cdot \frac{1}{1+\left(-\frac{1}{2}\right)} \cdot \left(x^2+2x\right)^{1-\frac{1}{2}} + C$$

$$= \frac{1}{2}\int f(g(x)) \cdot g'(x)\,dx$$

$$= \frac{1}{2} \cdot \frac{1}{\left(\frac{1}{2}\right)} \cdot \left(x^2+2x\right)^{\frac{1}{2}} + C$$

$$= \frac{1}{2} \cdot \frac{2}{1} \cdot \left(x^2+2x\right)^{\frac{1}{2}} + C$$

$$= \left(x^2+2x\right)^{\frac{1}{2}} + C$$

$$f(T) = T^{-\frac{1}{2}}$$

$$T = g(x)$$

$$(f\cdot g)' = f'g + fg'$$

$\cos^2 x$

$\boxed{\cos x \cdot \cos x}$

$$(\sin x \cdot \cos x)' = \cos x \cos x + \sin x(-\sin x)$$

$$\sin x \cos x = \boxed{\int \cos^2 x \, dx} + \int -\sin^2 x \, dx$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$\sin x \cos x = \int \cos^2 x \, dx - \int \sin^2 x \, dx$$

$$\sin x \cos x = \int \cos^2 x \, dx - \int (1 - \cos^2 x)\, dx$$

$$\sin x \cos x = \int \cos^2 x \, dx - \underbrace{\boxed{\int 1 \, dx}}_{x} + \int \cos^2 x \, dx$$

$$\sin x \cos x = -x + 2\int \cos^2 x \, dx$$

$$\Rightarrow \int \cos^2 x \, dx = \frac{1}{2}\left(x + \sin x \cos x\right) + C$$

$$= \frac{x}{2} + \frac{\sin 2x}{4} + C$$

cor

$\sin 2x =$

$2 \sin x \cos x$

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$x \quad \cos x \qquad\qquad 1\cdot\cos x + x\cdot\sin x$$

$$\int x\cdot\sin x\,dx$$

$$x\cos x = \int \cos x\,dx + \int x\sin x\,dx$$

$$x\cos x = \sin x + \int x\sin x\,dx$$

$$\int x\sin x\,dx = x\cos x - \sin x + C$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\cos x \cdot \cos x + \sin x \cdot (-\sin x)$$

$$\int \cos^2 x \, dx =$$

$$\int \cos x \cdot \cos x \, dx$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\left(\frac{1}{3}\cos^3(x)\cdot\frac{1}{\sin x}\right)' =$$

$$\left(\frac{1}{3}\cos^3(x)\right)'\cdot\frac{1}{\sin x} + \frac{1}{3}\cos^3(x)\cdot\left(\frac{1}{\sin x}\right)' =$$

$$\frac{1}{3}\cdot 3\cos^2(x)\cdot(-\cancel{\sin x})\cdot\frac{1}{\cancel{\sin x}} + \frac{1}{3}\cos^3(x)\cdot\frac{-1}{\sin^2 x}\cdot\cos x =$$

$$-\cos^2(x) - \frac{1}{3}\,\frac{\cos^4 x}{\sin^2 x}$$