$\boxed{\text{Soit } M \text{ une matrice carrée}}$

$$M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

$$\boxed{M' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}$$

V.p. de M' :

$1, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Big| \; 2, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$3, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$M' \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix} \overset{?}{=} \cancel{k \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

Soit u un vecteur propre de M. $\qquad M \in M_3(\mathbb{R})$

$\exists \lambda \in \mathbb{R}$ tq. $\quad M \cdot u = \lambda \cdot u$

$\Leftrightarrow \quad M \cdot u - \lambda \cdot u = 0$

matrice identité I_3

$$\lambda \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \lambda \cdot \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right] = \left[\lambda \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$\Leftrightarrow \quad M\cdot u - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \cdot u = 0$$

$$\Leftrightarrow \quad \left(M - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right) \cdot u = 0$$

$$\Leftrightarrow \quad \begin{pmatrix} m_{11}-\lambda & m_{12} & m_{13} \\ m_{21} & m_{22}-\lambda & m_{23} \\ m_{31} & m_{32} & m_{33}-\lambda \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\exists\, u \neq 0$ satisfaisant l'éq. ci-dessus

$$\Leftrightarrow \quad \underbrace{\det\left(M - \lambda I_3 \right)}_{\text{polynôme caractéristique de } M.} = 0$$

<u>Exemple:</u> $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ rel. à $\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$

Calculer le polynôme caractéristique de M:

$$\det \begin{pmatrix} 1-X & 2 \\ 3 & 4-X \end{pmatrix} = (1-X)(4-X) - 6$$

$$= x^2 - 5x + 4 - 6$$

$$= x^2 - 5x - 2 = 0 \iff x = \frac{5 \pm \sqrt{33}}{2}$$

$$x \simeq \begin{cases} 5{,}3 \\ -0{,}3 \end{cases}$$

$$\begin{tikzcd}
\mathbb{R}^2 \arrow[r, "M"] & \mathbb{R}^2 \arrow[d, "P^{-1}"] \\
\mathbb{R}^2 \arrow[u, "P"] \arrow[r] & \mathbb{R}^2
\end{tikzcd}$$

$$M' = P^{-1} M P$$

$$M' = \begin{pmatrix} 5{,}3 & 0 \\ 0 & -0{,}3 \end{pmatrix}$$

$$\boxed{\text{V.p. } x = 5{,}3}$$

$$\frac{5+\sqrt{33}}{2}$$

$$\ker \begin{pmatrix} 1-5{,}3 & 2 \\ 3 & 4-5{,}3 \end{pmatrix} = \ker \begin{pmatrix} -4{,}3 & 2 \\ 3 & -1{,}3 \end{pmatrix}$$

$$\begin{pmatrix} -4{,}3 & 2 \\ 3 & -1{,}3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2/4{,}3 \\ 1 & -1{,}3/3 \end{pmatrix} \sim \begin{pmatrix} 1 & -0{,}46 \\ 1 & -0{,}43 \end{pmatrix}$$

$$\begin{pmatrix} \left(1 - \frac{5+\sqrt{33}}{2}\right) & 2 \\ 3 & \left(4 - \frac{5+\sqrt{33}}{2}\right) \end{pmatrix} \sim \begin{pmatrix} \frac{-3-\sqrt{33}}{2} & 2 \\ 3 & \frac{3-\sqrt{33}}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \dfrac{4}{-3-\sqrt{33}} \\ 1 & \dfrac{3-\sqrt{33}}{6} \end{pmatrix} \sim \begin{pmatrix} 1 & \dfrac{3-\sqrt{33}}{6} \\ 0 & 0 \end{pmatrix}$$

$$\frac{-4}{3+\sqrt{33}} \cdot \frac{3-\sqrt{33}}{3-\sqrt{33}} = \frac{-4(3-\sqrt{33})}{9-33}$$

$$= \frac{-4(3-\sqrt{33})}{-24} = \frac{3-\sqrt{33}}{6}$$

$$x_1 + \frac{3-\sqrt{33}}{6} x_2 = 0 \quad \alpha$$

Ens. des sols: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k \boxed{\begin{pmatrix} \dfrac{\sqrt{33}-3}{6} \\ 1 \end{pmatrix}}$

Vect. propre: $u = \begin{pmatrix} \dfrac{\sqrt{33}-3}{6} \\ 1 \end{pmatrix}$

$ax + by + c = 0 \qquad \vec{d} = \begin{pmatrix} -b \\ a \end{pmatrix}$

$$\boxed{\text{V.p. } x = \frac{5-\sqrt{33}}{2}}$$

on déduit u

$$\begin{pmatrix} 1 - \dfrac{5-\sqrt{33}}{2} & 2 \\ 3 & 4 - \dfrac{5-\sqrt{33}}{2} \end{pmatrix} \sim \begin{pmatrix} \dfrac{-3+\sqrt{33}}{2} & 2 \\ 3 & \dfrac{3+\sqrt{33}}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{4}{-3+\sqrt{33}} \\ 1 & \frac{3+\sqrt{33}}{6} \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{3+\sqrt{33}}{6} \\ 0 & 0 \end{pmatrix}$$

$$\frac{4}{-3+\sqrt{33}} \cdot \frac{3+\sqrt{33}}{3+\sqrt{33}} = \frac{4(3+\sqrt{33})}{24} = \frac{3+\sqrt{33}}{6}$$

$$x_1 + \frac{3+\sqrt{33}}{6} x_2 = 0$$

Ens. des solutions : $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k \begin{pmatrix} \frac{3+\sqrt{33}}{6} \\ -1 \end{pmatrix}$

$\Rightarrow$ La matrice de M rel. à la base

$$\left(\begin{pmatrix} \frac{\sqrt{33}-3}{6} \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{33}+3}{6} \\ -1 \end{pmatrix} \right)$$

est diagonale et s'écrit : $\begin{pmatrix} \frac{5+\sqrt{33}}{2} & 0 \\ 0 & \frac{5-\sqrt{33}}{2} \end{pmatrix}$

$y'(3+4y)$

$$\int (3+4y(x)) \cdot y'(x)\, dx$$

$$\int (3+4t)\, dt$$

$$\boxed{3t+2t^2+C}$$

$$\int \frac{1}{4} \cdot (3+4t)^1 \cdot 4 \quad dt$$

$$\frac{1}{4} \cdot \frac{1}{2} \cdot (3+4t)^2$$

$$\frac{1}{8}(3+4t)^2$$