$\boxed{\text{TE OS}}$ Jeudi 12/12/2024

11 à 15 p. 35 à 36

1 page A4 de <u>théorie</u> (pas d'exemples)

But

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^5 \simeq 148{,}41 = 1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \dots$$

[Exemples p. 12]

1) $\lim\limits_{k \to \infty} \dfrac{u_{k+1}}{u_k} = C$

$$\sum_{k=1}^{\infty} u_k$$

série

$$\frac{1}{k!} = \frac{1}{k \cdot (k-1) \cdot (k-2) \cdot (k-3) \cdot \ldots \cdot 1} = u_k$$

u_k est une suite

$$\frac{1}{(k+1)!} = \frac{1 \cdot \boxed{1}}{(k+1) \cdot \boxed{k \cdot (k-1) \cdot \ldots \cdot 1}} = U_{k+1} = \frac{1}{k+1} \cdot u_k$$

$$\sum_{k=1}^{\infty} \frac{1}{k!} = \ell$$

$$\frac{U_{k+1}}{u_k} = \frac{\frac{1}{k+1} \cdot \frac{1}{k!}}{\frac{1}{k!}} = \frac{1}{k+1} \xrightarrow{k \to \infty} 0$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{k!} \quad \text{converge}$$

$$\frac{u_{k+1}}{u_k} = \frac{\frac{1}{k+1} \cdot \cancel{u_k}}{\cancel{u_k}} = \frac{1}{k+1}$$

2)

$$u_k = \frac{2^k}{k}$$

$$u_{k+1} = \frac{2^{k+1}}{k+1}$$

$$\frac{2^{k+1}}{2^k} = \frac{2^k \cdot 2^1}{2^k} = 2$$

$$\frac{u_{k+1}}{u_k} = \frac{\frac{2^{k+1}}{k+1}}{\frac{2^k}{k}} = \frac{2^{k+1}}{k+1} \cdot \frac{k}{2^k} = 2 \cdot \frac{k}{k+1}$$

$$\frac{k}{k+1} \xrightarrow{k\to\infty} 1$$

$$2 \cdot \frac{k}{k+1} \xrightarrow{k\to\infty} 2 \cdot 1 = 2 > 1$$

$$\Rightarrow \lim_{k\to\infty} \frac{u_{k+1}}{u_k} = 2 > 1$$

$\Rightarrow$ La série $\sum u_k$ diverge (Critère du quotient)

$$u_k = u(k) = \frac{1}{k!}$$

$$u(k+1) = \frac{1}{(k+1)!} = \frac{1}{(k+1)\cdot k\cdot(k-1)\cdot\ldots\cdot 1}$$

$$u_k = u(k) = \frac{2^k}{k} = \frac{2^{(k+1)}}{(k+1)} = \frac{2^{k+1}}{k+1}$$

$$\frac{\frac{k+1}{k+2}}{\frac{k}{k+1}} = \frac{k+1}{k+2}\cdot\frac{k+1}{k} = \frac{u_{k+1}}{u_k}$$

$$= \frac{k^2+2k+1}{k^2+2k} \xrightarrow{k\to\infty} \frac{k^2}{k^2} \xrightarrow{k\to\infty} 1$$

$$\lim_{k\to\infty} \frac{u_{k+1}}{u_k} = 1 \implies$$ Le critère ne donne pas d'info.

(On sait déjà, par la propriété 3 p 5, que $\sum \frac{k}{k+1}$ diverge.)

$$\lim_{x \to \infty} \frac{P(x)}{Q(x)} = \lim_{x \to \infty} \frac{a_n x^n}{b_n x^p} = \lim_{x \to \infty} \frac{a_n}{b_n} \cdot x^{n-p}$$

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

$$Q(x) = b_p x^p + b_{p-1} x^{p-1} + \cdots + b_0$$

$$\lim_{k\to+\infty} \frac{\sin\left(\frac{1}{k}\right)}{\frac{1}{k}} = \lim_{x\to 0} \frac{\sin x}{x} \overset{\text{B.-H.}}{=} 1$$

$$x = \frac{1}{k}$$

Equivalentes, d'après le critère

$$\sum \sin \frac{1}{k} \sim \sum \frac{1}{k}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = « \frac{0}{0} »$$

$$\frac{(\sin x)'}{x'} = \frac{\cos x}{1} = \cos x \xrightarrow{x\to 0} 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \cos x = 1$$

$$\frac{1}{k(k+1)} = \frac{1}{k^2+k} = u_k \qquad k \geq 1$$

$k \geq 1 \qquad k^2+k > k^2$

$$u_k = \frac{1}{k^2+k} < \frac{1}{k^2} = v_k$$

$\Rightarrow \quad u_k \geq 0$ et $u_k < v_k \quad \forall\, k \geq 1$

$\Rightarrow \quad \sum u_k$ converge car $\sum v_k$ est de Riemann

par le critère de comparaison p. 9

$$\frac{u_k}{v_k} = \frac{k+3}{k^3-12k+7} \cdot \frac{k^2}{1}$$

$$= \frac{k^3+3k^2}{k^3-12k+7} \xrightarrow{k\to\infty} \frac{k^3}{k^3} = \frac{1}{1} \xrightarrow{k\to\infty} 1$$

$\Rightarrow$ u_k est équiv. à v_k

Example 1 p. 114:

$$\int_p^{+\infty} \frac{1}{x\ln(x)}\,dx$$

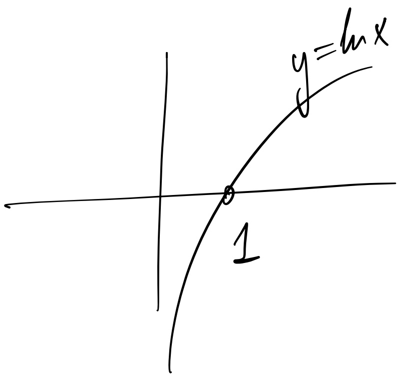
$$f(x) = \frac{1}{x\ln(x)}$$

$$D_f =]0;+\infty[- \{1\}$$
$$=]0;1[\cup]1;+\infty[$$

$x\ln(x) = 0 \not\Leftrightarrow x = 0$

$\ln(x) = 0 \,/\, x = 1$

$$\boxed{p=2}$$



$$\int_2^{+\infty} \frac{1}{x\ln(x)}\,dx = \int_2^{+\infty} \ln(x)^{-1}\cdot\frac{1}{x}\,dx = \int_2^{+\infty} (\ln x)^{-1}\cdot(\ln x)'\,dx$$

$\frac{1}{x}\cdot\frac{1}{\ln x}$ $\quad$ $\frac{1}{x} = \ln(x)'$

$$\boxed{\int T^{-1}\,dT = \ln(T) + c}$$

$$= \ln(\ln(x))\Big|_2^{+\infty}$$

$$= \lim_{t\to\infty} \ln(\ln(x))\Big|_2^{t} = \lim_{t\to\infty} \ln(\ln(t)) - \ln(\ln(2))$$

$$= +\infty$$