H est la matrice de h dans la base canonique.

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{H} & \mathbb{R}^2 \\ P \uparrow & & \downarrow P^{-1} \\ \mathbb{R}^2 & \xrightarrow{H'} & \mathbb{R}^2 \end{array}$$

$$H = \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$$

$$P = \begin{pmatrix} -2 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 1-X & 2 \\ -3 & -6-X \end{vmatrix} = (X-1)(X+6) + 6$$

$$= X^2 + 6X - X - 6 + 6$$

$$= X^2 + 5X = X(X+5)$$

$$H' = P^{-1} H P = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix}$$

$\lambda_1 = 0 \quad \lambda_2 = -5$

$$E_0 : \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \quad \begin{cases} x_1 = -2x_2 \\ x_2 = x_2 \end{cases}$$

$$E_0 = \left\{ k \begin{pmatrix} -2 \\ 1 \end{pmatrix} \mid k \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\rangle$$

Base de E_0 : $\left(\begin{pmatrix} -2 \\ 1 \end{pmatrix} \right)$

Base dans laquelle H' est diagonale :

$$\left(\begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right)$$

$$E_{-5} : \begin{pmatrix} 6 & 2 \\ -3 & -1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{matrix} x_1 = -\frac{1}{3} x_2 \\ x_2 = x_2 \end{matrix}$$

$$E_{-5} = \left\{ k \begin{pmatrix} -1 \\ 3 \end{pmatrix} \mid k \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right\rangle$$

Base de E_{-5} : $\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} \right)$

$$H' = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix}$$

image de e_1

$$\begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

image de e_2

$$\begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

image de e_1'

$$\begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = h(-2;1)$$

image de e_2'

$$\begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -15 \end{pmatrix} = -5\begin{pmatrix} -1 \\ 3 \end{pmatrix} = h(-1;3)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = x\begin{pmatrix} -2 \\ 1 \end{pmatrix} + y\begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$h(x;y) = x \cdot \overbrace{h(-2;1)}^{0} + y \cdot \underbrace{h(-1;3)}_{-5\cdot(-1;3)}$$

$$= -5y \cdot (-1;3)$$

$$h(x;y) = (0; -5y) = (0x+0y; 0x-5y) = (0; -5y)$$

$$H' = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix}$$