$y, y', x, f(x), \ldots$

$$g(y) \cdot y' = f(x)$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$x \cdot y' = \frac{x-1}{y}$$

$$y y' = \frac{x-1}{x} \Leftrightarrow y \cdot \frac{dy}{dx} = \frac{x-1}{x}$$

$$x y' = \frac{x}{y} - \frac{1}{y}$$

$$y\,dy = \frac{x-1}{x}\,dx$$

$$\boxed{\frac{1}{y} + x y' = \frac{x}{y}}$$

$$1 - \frac{1}{x}$$

$$\int y\,dy = \int \frac{x-1}{x}\,dx$$

$$\underbrace{\frac{1}{2}y^2}_{G(y)} = x - \ln|x| + c$$

équation implicite

$$y = \pm\sqrt{2(x-\ln|x|)+c}$$

$$(*)\quad \boxed{y' \cdot (1+x^2) = xy}$$

$y=0$ est solution de $(*)$

$$\frac{y'}{y} = \frac{x}{1+x^2}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{x}{1+x^2}$$

$$\frac{1}{y} \cdot dy = \frac{x}{1+x^2} \cdot dx$$

$$\ln|y| = \int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + C$$

$$r\ln(a) = \ln(a^r)$$

$$\ln|y| = \ln\left(\sqrt{x^2+1}\right) + c$$

$e^{(\,)}$

$$e^{\ln|y|} = e^{\left(\ln\left(\sqrt{x^2+1}\right) + c\right)}$$

$$e^{\ln(a)} = a$$

$$= e^{\ln\left(\sqrt{x^2+1}\right)} \cdot \underbrace{e^{c}}_{k>0}$$

$$e^{a+b} = e^{a} \cdot e^{b}$$

$$|y| = k \cdot \sqrt{x^2+1}$$

$$y = \pm k \cdot \sqrt{x^2+1} \qquad k>0$$

$$\Leftrightarrow \boxed{y = k\sqrt{x^2+1} \qquad k \in \mathbb{R}}$$

$$y(0) = 1 \Rightarrow k = 1$$

$$\boxed{y' - \frac{y}{x} = x} \quad (*) \qquad \cdot e^{\int (-\frac{1}{x})\,dx}$$

Sol gen. de $(**)$

$$(**) \quad \boxed{y' - \frac{y}{x} = 0}$$

$$y' + a(x)\cdot y = 0$$

$$\frac{y'}{y} = -a(x) \Rightarrow y = k\cdot e^{-\int a(x)\,dx}$$

$$\frac{y'}{y} = \frac{1}{x} \qquad \ln|y| = \ln|x| + C$$

$$y = k\cdot x$$

$$e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = \frac{1}{|x|} \qquad e^{\int a(x)\,dx}$$

choisir $\frac{1}{x}$

$$\boxed{\underbrace{y'\cdot e^{-\ln|x|}}_{u'v} \underbrace{- \frac{1}{x}\cdot y\cdot e^{-\ln|x|}}_{+uv'}} = x\cdot\frac{1}{|x|}$$

Je choisis 1

$$\underbrace{\left(y\cdot e^{-\ln|x|}\right)'}_{(u\cdot v)} = x\cdot e^{-\ln|x|} = \pm 1$$

$$\int$$

$$y \cdot \underbrace{e^{-\ln|x|}}_{\frac{1}{x}} = x \leftarrow \text{choix}$$

$$e^{-2} = \frac{1}{e^2}$$

$$y = x^2$$

Solution de (*) :

$$\boxed{y = \underbrace{k \cdot x}_{\text{sol. gén. de l'éq. hom. (**)}} + \underbrace{x^2}_{\text{sol. part. de l'éq. (*)}}}$$

linéaire du 1er ordre

$$\boxed{y' + a(x) \cdot y = g(x)}$$

$$\boxed{\text{Facteur intégrant : } e^{\int a(x)\,dx}}$$

$$\left(y' \cdot e^{\int a(x)dx}\right) + \left(y \cdot a(x) \cdot e^{\int a(x)dx}\right) = g(x)$$

$$\left(y \cdot e^{\int a(x)dx}\right)' = g(x)$$