Calculer $f(1{,}5)$ si $f(x) = \cos(x)\ln(x)$ $D_f :]0;+\infty[$

Opérations autorisées : $+$, $-$, $\div$, $\cdot$

Taylor : Au voisinage de 1 ; $\pi/4$

$]0;2[\subset D_f$ inclus dans

$$f(x) = f(1) + f'(1)\cdot(x-1) + f''(1)\cdot(x-1)^2 \cdot \frac{1}{2!} + f'''(1)\cdot(x-1)^3 \cdot \frac{1}{3!} + R_3(x)$$

$$(\cos x \ln x)' = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}$$

$$f'(1) = \boxed{\cos(1)} \simeq 0{,}54$$

$$(\cos x \ln x)'' = -\cos x \cdot \ln x - \sin x \cdot \frac{1}{x} + (-\sin x)\cdot\frac{1}{x} + \cos x \cdot\left(-\frac{1}{x^2}\right)$$

$$= -\cos x \ln x - \frac{2\sin x}{x} - \frac{\cos x}{x^2}$$

$$f''(1) = \boxed{-2\sin(1) - \cos(1)} \simeq -2{,}22$$

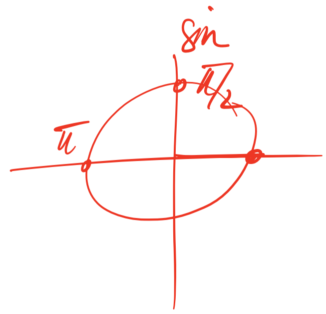
$$(\cos x \ln x)''' = -\sin x \cdot \ln x - \cos x \cdot \frac{1}{x} - 2\left(\frac{\cos x}{x} + \sin x \cdot\left(-\frac{1}{x^2}\right)\right)$$

$$-\left(\frac{-\sin x}{x^2} + \cos x \cdot (-2)\cdot\frac{1}{x^3}\right)$$

$$= -\sin x \ln x - \frac{3\cos x}{x} + \frac{2\sin x}{x^2} + \frac{\sin x}{x^2} + \frac{2\cos x}{x^3}$$

$$f'''(1) = -3\cos(1) + 2\sin(1) + \sin(1) + 2\cos(1)$$

$$= \boxed{3\sin(1) - \cos(1)} \simeq 1{,}98$$



$$\sin : \mathbb{R} \to \mathbb{R}$$

$$x \mapsto \sin x$$

en radians

$$\sin(0) = 0$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin(\pi) = 0$$

$$\cos(x)\ln(x) \cong 0 + 0{,}54\cdot(x-1) - 2{,}22\cdot\frac{(x-1)^2}{2} + 1{,}98\cdot\frac{(x-1)^3}{6}$$

$$\cong 0{,}54(x-1) - 1{,}11\cdot(x-1)^2 + 0{,}33\cdot(x-1)^3 \quad + R_3(x)$$

$$+ R_3(x)$$

$$\cos(x) = \cos(0) + (-\sin(0)) \cdot x + (-\cos(0)) \cdot \frac{x^2}{2!} + \sin(0) \cdot \frac{x^3}{3!} + \cos(0) \cdot \frac{x^4}{4!} + (-\sin(0)) \cdot \frac{x^5}{5!} + R_5(x)$$

$I =]-\frac{1}{2} ; \frac{1}{2}[$

$$\boxed{\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + R_5(x)}$$

$$\underbrace{R_5(x) = \frac{-\cos(c)}{6!} \cdot x^6}_{\text{formule de Taylor}}$$

$$|R_5(x)| \leq \frac{|x|^6}{6!} \cdot \sup_{t \in I} |-\cos(t)| \quad 1$$

$$\Rightarrow |R_5(x)| \leq \frac{|x|^6}{720} \quad \text{avec } x \in]-\tfrac{1}{2} ; \tfrac{1}{2}[$$

$$\Rightarrow |R_5(x)| \leq \frac{1}{64 \cdot 720} \simeq 0{,}000022$$

$$\sup_{t \in]-\frac{1}{2} ; \frac{1}{2}[} |-\cos(t)| = \sup_{t \in]-\frac{1}{2} ; \frac{1}{2}[} |\cos(t)| = 1$$

car $\cos(0) = 1$ et c'est le max de cos.